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Discontinuous Galerkin scheme for the spherical shallow water equations with applications to tsunami modeling and prediction

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ABSTRACT

We present a novel high-order discontinuous Galerkin discretization for the spherical shallow water equations, able to handle wetting/drying and non-conforming, curved meshes in a well-balanced manner. This requires a well-balanced discretization, that cannot rely on exact quadrature, due to the curved mesh. Using the strong form of the discontinuous Galerkin discretization, we achieve a splitting of the well-balanced condition into individual problems for the flux and volume terms, which has significant advantages: It allows for the construction of non-conforming, well-balanced flux discretizations, i.e. we can perform non-conforming mesh refinement while preserving the well-balanced property of the scheme. More importantly, this approach enables the development of a new method for handling wet/dry transitions. In contrast to other wetting/drying methods, it is well-balanced and able to handle wetting/drying robustly at any polynomial order, without the introduction of physical model assumptions such as viscosity, artificial porosity or cancellation of gravity.

We perform a series of one-dimensional tests and analyze the properties of our scheme. In order to validate our method for the simulation of large-scale tsunami events on the rotating sphere, we perform numerical simulations of the 2011 Tohoku tsunami and compare our results to real-world buoy data. The method is able to predict arrival times and wave amplitudes accurately even over long distances. This indicates that our method accurately captures all physical phenomena relevant to the long-term evolution of tsunami waves.

1. Introduction

Numerical methods for the simulation of the shallow water equations have seen a considerable amount of research interest during the last decades, as they can be used to model a wide variety of physical phenomena from storm surges to tsunami propagation. One of the long-standing goals is the development of a tsunami early warning system for hazard

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forecast and risk assessment. This requires a physically accurate model and a robust and efficient numerical method for the simulation of the tsunami propagation.

The simulation of large-scale tsunami events pose some challenges for the physical model and the numerical method. On the model side, it is crucial to take interaction with land masses and effects of Earth's curvature into account. Tsunami events affect oceans globally, thus for realistic simulations at large scales, we have to incorporate curvature of the sphere, wetting/drying and the Coriolis force into the model.

The numerical method on the other hand has to be adaptive, robust and highly efficient. A particularly promising numerical method is the discontinuous Galerkin (dG) method, as it offers numerous advantages over competing methods for the simulation of wave propagation problems. Among them are high-order accuracy, innate parallelism and the flexibility of *hp*-adaptivity [1]. Additionally, for the simulation of tsunamis we require the well-balanced property and the ability to handle wetting and drying accurately. Since tsunamis can be regarded as perturbations of the water at rest steady state, it is important to use well-balanced schemes, which can conserve the steady state numerically [2–4]. Flooding and drying is important as tsunami waves interact with land masses and capturing the correct, physical behavior at the wet/dry transition is crucial for the accurate simulation of tsunamis.

In this work, we present a novel method for the simulation of the shallow water equations on the surface of a rotating sphere using the discontinuous Galerkin method. This requires the development of a high-order dG method, which can handle curved, non-conforming meshes with discontinuous bottom topography and wet/dry transitions in a well-balanced manner.

Although well-balanced schemes have received considerable attention, there appears to be only limited past work considering well-balanced discontinuous Galerkin schemes on curved meshes. The key difficulty is the lack of exact numerical integration on which many of the well-balanced methods rely on [5–8]. Notable exceptions are presented by Chandrashekar and Zenk [9] and Wintermeyer et al. [10]. In both papers, the authors exploit the strong form of the discontinuous Galerkin formulation to match the discretizations of flux and source terms and construct a well-balanced scheme in this way. In the latter, a well-balanced and entropy-stable dG scheme is presented for the shallow water equations on curvilinear elements. However, no discussion of non-conforming meshes or wetting/drying is included.

The inclusion of land masses poses the question of how to treat the shoreline in the discontinuous Galerkin context. One approach is to handle shorelines as free boundaries [11], which is in line with the physical model as it loses its validity in dry areas. This has the advantage that numerical problems, associated with dry areas in the solution, are avoided. On the other hand, an accurate model for the dynamics of the shoreline is required in order to calculate the position of the moving boundary. Often, constant remeshing is required and topological changes need to be taken care of, amounting to prohibitive computational costs. An alternative adopted in numerous works is to handle wetting and drying within the computational domain. These approaches can be seen as a special type of immersed boundary method. Wet/dry transitions introduce three distinct numerical difficulties: maintaining positivity of the water height, introduction of unphysical pressure gradients, and numerical instabilities due to the discontinuities associated with the transition. Various methods exist to ensure positivity of the approximate solution [7,12,13]. A popular method introduced by Xing et al. [12] is to maintain the positivity of cell-averages using a timestep restriction. Positivity on the nodes is then ensured by using a positivity-preserving limiter, which rescales the polynomial around the positive average. While it is claimed that this method is well-balanced [12,14], it is not unconditionally well-balanced as partly dry cells are neglected. Semi-dry cells introduce artificial gradients and generate unphysical waves at the shores. This effect, also called "numerical storms" sometimes [10], has been observed by other authors, but there exists no satisfactory solution to this problem. Kesserwani and Liang [8] propose a reconstruction of nodal values such that the pressure gradients vanish for the "lake at rest" solution. The authors present this method for a piecewise linear method in one dimension and it is unclear how this approach performs for higher-order methods. Bunya et al. [15] as well as [6] propose to cancel gravity in these cells to eliminate the problem of artificial pressure gradients. This requires the introduction of dual-valued fluxes in order to make the scheme well-balanced for the case of the "lake at rest" solution. While both wet/dry treatments make the scheme well-balanced, they are not consistent with the physical model at the shores and appear to be restricted to piecewise linear polynomials. Other approaches use "artificial porosity" and introduce a fraction indicator to represent how much of the cell is wet and how much is dry [16,17]. This allows for implicit time integration with large steps but introduces other problems such as higher wave speeds in the wet/dry region and an altered shallow water model. Finally there is the issue of stability at the wet/dry interface. Most previously mentioned wetting/drying algorithms reduce the order of the solution to linear polynomials and apply a slope limiter to prevent unphysical discharges [12,15]. Meister and Ortleb [7] use an implicit scheme with a modal filter and a shock indicator to stabilize the scheme in the nearly dry regions.

The numerical solution of the shallow water equations on the sphere using the discontinuous Galerkin method has been investigated by various authors, although typically neglecting the influence of bottom topography [18,19]. Blaise and St-Cyr [20] construct a discontinuous Galerkin method on the sphere for tsunami simulations and use an adjoint method in order to optimize initial conditions based on buoy data comparison [21]. This method is promising as it demonstrates the feasibility of using discontinuous Galerkin simulations on the sphere in combination with an adjoint method to reconstruct initial conditions from buoy data to obtain accurate early warnings. Their method lacks wetting/drying however, and shores are treated as reflecting boundaries, which result in unphysical reflections at the shorelines.

In this paper, we propose a general method for the construction of well-balanced dG discretizations on curved, non-conforming meshes with wet/dry transitions. Our method is based on the observation that the condition for a well-balanced