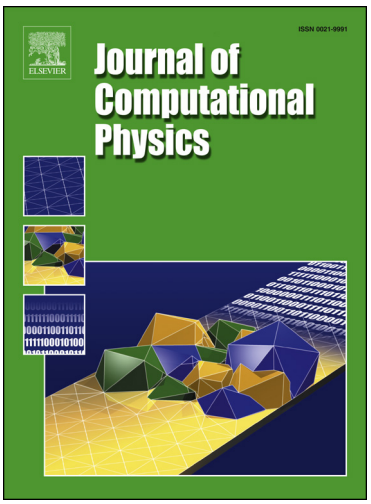
Accepted Manuscript



Artificial viscosity to cure the carbuncle phenomenon: the three-dimensional case

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PII: S0021-9991(18)30077-9
DOI: https://doi.org/10.1016/j.jcp.2018.02.001
Reference: YJCPH 7839

To appear in: *Journal of Computational Physics*

Received date: 28 September 2017
Revised date: 18 January 2018
Accepted date: 2 February 2018

Please cite this article in press as: A.V. Rodionov, Artificial viscosity to cure the carbuncle phenomenon: the three-dimensional case, *J. Comput. Phys.* (2018), https://doi.org/10.1016/j.jcp.2018.02.001

Artificial viscosity to cure the carbuncle phenomenon: the three-dimensional case

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Keywords: Carbuncle phenomenon· Shock instability · Godunov-type methods · Riemann solvers · Artificial viscosity

1. Introduction

The *carbuncle phenomenon* (also known as the *shock instability*) has remained a serious computational challenge since it was first noticed and described [1, 2]. In [3] the author presented a summary on this subject and proposed a new technique for curing the problem. Its idea is to introduce some dissipation in the form of right-hand sides of the Navier-Stokes equations into the basic method of solving Euler equations; in so doing, the molecular viscosity coefficient is replaced by the artificial viscosity coefficient. The new cure for the carbuncle flaw was tested and tuned for the case of using first-order schemes in two-dimensional simulations. Its efficiency was demonstrated on several well-known test problems. In this paper we extend the technique of [3] to the case of three-dimensional simulations.

2. Governing equations and space discretization

The Euler equations. The governing equations for the compressible gas dynamics, written in Cartesian coordinates *xyz*, are

$$\frac{\partial \mathbf{U}}{\partial t}+\frac{\partial \mathbf{F}_x}{\partial x}+\frac{\partial \mathbf{F}_y}{\partial y}+\frac{\partial \mathbf{F}_z}{\partial z}=\mathrm{RHS},\qquad \mathbf{U}=\begin{bmatrix}\rho\\ \rho u_x\\ \rho u_y\\ \rho u_z\\ \rho h_0-P\end{bmatrix},\qquad \mathbf{F}_m=\begin{bmatrix}\rho u_m\\ \rho u_x u_m+\delta_{mx}P\\ \rho u_y u_m+\delta_{my}P\\ \rho u_z u_m+\delta_{mz}P\\ \rho h_0 u_m\end{bmatrix},\qquad m=x,y,z, \tag{1}$$

where RHS = 0 for the Euler equations, δ_{mn} is the Kronecker delta, $\mathbf{u} = (u_x, u_y, u_z)$ are the velocity vector and its components, ρ is the density, P is the pressure and h_0 is the specific total enthalpy, which for a polytropic gas is

$$h_0=\frac{1}{2}\left(u_x^2+u_y^2+u_z^2\right)+h,\quad h=\frac{\gamma P}{(\gamma-1)\rho},$$

with h denoting the specific enthalpy and γ denoting the ratio of specific heats.

The space discretization. We apply the finite volume approach to solve (1) numerically. In so doing we consider a sufficiently smooth structured grid and introduce curvilinear coordinates $\xi\eta\zeta$ that transform the grid in physical space *xyz* to a rectangular grid in computational space with the grid spacing $\Delta\xi = \Delta\eta = \Delta\zeta = 1$. The following geometric parameters of the grid will take the place of metrics (hereinafter the grid indices i, j and k correspond to the coordinates ξ, η and ζ): $V_{i,j,k}$, the cell volume; $(\mathbf{S}_\xi)_{i+½,j,k}$, $(\mathbf{S}_\eta)_{i,j+½,k}$ and $(\mathbf{S}_\zeta)_{i,j,k+½}$, the area vectors of the cell faces in three grid directions (each area vector points in the direction of increase in the associated coordinate).

Assume that the solution at a given time level n is defined, meaning that for the time $t = t^n$ we know the average values $\mathbf{Q}^n_{i,j,k}\equiv\left(u_x,u_y,u_z,\rho,P\right)^n_{i,j,k}$ within each cell. In order to update the solution