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Simulations of viscous and compressible gas–gas flows using high-order finite difference schemes

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ABSTRACT

A computational method for the simulation of viscous and compressible gas–gas flows is presented. It consists in solving the Navier–Stokes equations associated with a convection equation governing the motion of the interface between two gases using high-order finite-difference schemes. A discontinuity-capturing methodology based on sensors and a spatial filter enables capturing shock waves and deformable interfaces. One-dimensional test cases are performed as validation and to justify choices in the numerical method. The results compare well with analytical solutions. Shock waves and interfaces are accurately propagated, and remain sharp. Subsequently, two-dimensional flows are considered including viscosity and thermal conductivity. In Richtmyer–Meshkov instability, generated on an air–SF6 interface, the influence of the mesh refinement on the instability shape is studied, and the temporal variations of the instability amplitude is compared with experimental data. Finally, for a plane shock wave propagating in air and impacting a cylindrical bubble filled with helium or R22, numerical Schlieren pictures obtained using different grid refinements are found to compare well with experimental shadowgraphs. The mass conservation is verified from the temporal variations of the mass of the bubble. The mean velocities of pressure waves and bubble interface are similar to those obtained experimentally.

1. Introduction

Gas–gas flows with strong effects of compressibility are encountered in various areas, from astrophysics, such as in supernova expansion where the fluid mixing is attributed to Richtmyer–Meshkov instability [6], to engineering, such as in supersonic combustion [45]. A main challenge for the simulation of these flows is that a deformable interface separates two fluids of different properties; furthermore, other discontinuities such as shock waves may arise. Specific concerns are that the computational methods capture such discontinuities accurately, without introducing spurious perturbations, and the accurate conservation of mass of the fluids.

Several methods have been developed for the simulation of a deformable interface [44,55]. These methods can be classified into interface-tracking and interface-capturing methods. Interface-tracking methods use an adaptive mesh, some points of which coincide with an interface [22,29,62], such that a mesh moves with the interface, possibly in conjunction with a uniform background mesh. These methods can be accurate, but difficulties arise during interface rupture or coalescence

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events, and implementation is involved for a three-dimensional geometry. In interface-capturing methods, a fixed mesh can be used. These methods simulate interface evolution by solving an advection equation for a scalar field.

A commonly-used type of interface-capturing methods is the level-set method [28,47,48], wherein the scalar field corresponds to the signed distance to the nearest interface. A drawback of that method is that, in order to maintain the level-set function as a signed-distance function, which is desirable for accurate determination of the normal vector of an interface, a reinitialisation step is commonly used, which may introduce significant mass errors. But some modern level-set methods appear not to be prone to this [58]. For compressible multi-component flows, level-set methods are frequently used in conjunction with a ghost-fluid method to apply interface conditions (e.g., [48]). It seems unclear, however, how such an approach could be generalized to simulate flows for viscous/conductive fluids, which is our objective here, as imposing the corresponding stress/energy jump conditions at interfaces would require further development, although such a formulation is available for incompressible flows [37].

An interesting alternative is to model an interface as a thin interfacial layer. Several approaches of this type have been developed for compressible flows. One of these is to use a single set of balance equations for a mixture; jumps in the value of fluid properties (such as those in the equation of state) occur in an interfacial layer. The system is then closed by an advection-type equation for (a function of) such a fluid property, or several of these, which can be put into conservative form upon combination with the mass balance equation for the entire mixture. The advected quantity is chosen such that interface conditions are satisfied accurately without introducing spurious oscillations in the pressure or in the fluid properties, when considering basic one-dimensional test problems. In particular, a judicious choice of an advected parameter reduces significantly deviations from a uniform pressure across an interface in a one-dimensional inviscid uniform flow; such pressure oscillations are specific to these multi-component systems (e.g., [1]). For the perfect gas equation of state, one advected quantity may be used (e.g., [1,2]). In all, a set of mixture relations used in the interfacial region should be consistent: that is, the primary variables and parameters in the equation of state should be determined in a consistent manner.

A second such approach is to resolve separately the balance equations for mass conservation, possibly also energy and momentum balance equations for each constituent. To obtain the local value of parameters in the equation of state inside an interfacial layer, these balance equations may have to be supplemented by a balance equation for the volume fraction of one of the constituents again such as to satisfy interface conditions in basic test problems (e.g., [4,15,54]). The formulation of an advection-type equation for the volume fraction is not straightforward however, and may require making assumptions (e.g., [34]). An approach based entirely on mass fractions has also been formulated [53].

This ‘diffuse’-interface type of model seems more suitable for systems with viscosity, wherein a discontinuity in tangential velocity does not have to be contended with. Accounting for conduction, and possibly for dependencies on temperature in the equation of state, requires reduction of errors in the temperature at basic one-dimensional tests, in addition to pressure errors, which has prompted further consideration of the formulation of mixture relations in view of errors in the temperature at basic one-dimensional tests, in addition to pressure errors (e.g., [7,34]).

These models have been formulated thus far mostly in a finite-volume methodology, in particular, using high-order WENO schemes. The objective of the present study is to develop high-order finite-difference methods for the simulation of viscous and compressible multi-component flows. These methods have initially been developed for aeroacoustic computations in order to provide negligible dissipation and dispersion errors [8,9]. In this work, the methods are adapted to the simulation of multi-component flows. High-order explicit centred finite difference schemes are used for spatial differentiation. In order to remove grid-to-grid oscillations, whose wavelength is equal to twice the mesh spacing, a centred selective filter is applied every time step, throughout the entire domain. A discontinuity-capturing methodology enables to capture various discontinuities such as deformable interfaces between two fluids or shock waves. The methods are employed to solve the Navier–Stokes equations associated with one advection equation governing the interface displacements [2,33]. In order to validate the present methods, three one-dimensional test cases are solved. The numerical results are compared with analytical solutions. Finally, the algorithm is employed to simulate two-dimensional viscous flows. Firstly, a Richtmyer–Meshkov instability generated on a post-shocked interface between air and sulphur hexafluoride (SF6) is investigated. Secondly, a plane shock wave propagating through air, impacting a cylindrical bubble filled with helium or chlorodifluoromethane (R22) gas is studied. The numerical results are compared with experimental solutions [24,30].

The present paper is organized as follows. The equations governing the test problems are given in Section 2. The numerical methods are detailed in Section 3. The one-dimensional and two-dimensional flows are presented, and solved in Sections 4 and 5, respectively. Concluding remarks are finally provided in Section 6.

2. Governing equations

2.1. Balance equations

Each of the two fluids considered does not occupy all space, so we shall make use of the volume fraction ϕ_α defined as that part of an infinitesimal volume occupied by fluid α, and denote by ρ_α the mass per unit volume of pure fluid α, and by e_α the internal energy per unit mass of pure fluid α. Pursuing a formulation of balance equations for an entire mixture, we note that the mixture density and internal energy are then