Journal of Computational Physics 360 (2018) 120–136

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

A high-order multiscale finite-element method for time-domain acoustic-wave modeling

Kai Gao [a], Shubin Fu [b,*], Eric T. Chung [b]

[a] *Geophysics Group, Los Alamos National Laboratory, Los Alamos, NM 87545, USA*
[b] *Department of Mathematics, The Chinese University of Hong Kong, Hong Kong*

A R T I C L E I N F O

Article history:
Received 10 September 2017
Received in revised form 12 January 2018
Accepted 17 January 2018
Available online xxxx

Keywords:
Acoustic-wave equation
Multiscale method
Finite-element method
Spectral-element method
Heterogeneous medium

A B S T R A C T

Accurate and efficient wave equation modeling is vital for many applications in such as acoustics, electromagnetics, and seismology. However, solving the wave equation in large-scale and highly heterogeneous models is usually computationally expensive because the computational cost is directly proportional to the number of grids in the model. We develop a novel high-order multiscale finite-element method to reduce the computational cost of time-domain acoustic-wave equation numerical modeling by solving the wave equation on a coarse mesh based on the multiscale finite-element theory. In contrast to existing multiscale finite-element methods that use only first-order multiscale basis functions, our new method constructs high-order multiscale basis functions from local elliptic problems which are closely related to the Gauss–Lobatto–Legendre quadrature points in a coarse element. Essentially, these basis functions are not only determined by the order of Legendre polynomials, but also by local medium properties, and therefore can effectively convey the fine-scale information to the coarse-scale solution with high-order accuracy. Numerical tests show that our method can significantly reduce the computation time while maintain high accuracy for wave equation modeling in highly heterogeneous media by solving the corresponding discrete system only on the coarse mesh with the new high-order multiscale basis functions.

© 2018 Elsevier Inc. All rights reserved.

1. Introduction

Accurate full-wavefield simulation is an important task in such as acoustics, electromagnetics and seismological. In exploration seismology for instance, full-wavefield-based imaging and inversion methods, including reverse-time migration (e.g., [23,2]), least-square reverse-time migration (e.g., [6,34,8]) and full waveform inversion (e.g., [28,29]), are becoming mainstream methods for modern seismic data processing and subsurface medium property characterization. The accuracy and efficiency of subsurface geological structure delineation and oil and gas reservoir characterization therefore largely depend on how accurately and efficiently the full-wavefield modeling problem can be solved.

Existing methods for time-domain full-wavefield modeling in heterogeneous media include finite-difference methods (e.g., [31,32,21]), pseudo-spectral methods [9], finite-element methods (e.g., [24,17,13]), spectral-element methods (e.g., [26,18,19]), phononic lattice and lattice Boltzmann methods [16,33], etc. The finite-difference methods and spectral-element methods are particularly widely applied because of their efficiency.

* Corresponding author.
E-mail addresses: kaigao87@gmail.com (K. Gao), shubinfu89@gmail.com (S. Fu), tschung@math.cuhk.edu.hk (E.T. Chung).

https://doi.org/10.1016/j.jcp.2018.01.032
0021-9991/© 2018 Elsevier Inc. All rights reserved.

However, these existing methods share a common difficulty in modeling wavefields for large-scale and highly heterogeneous models: the computation cost is directly proportional to the number of grids in the model. In 3D case, the issue of computational cost becomes even more severe. In seismological applications, for instance, modern seismic exploration tends to investigate increasingly finer details of target reservoirs or to characterize smaller and smaller geological heterogeneities. The number of grid points in these models therefore can become fairly large under certain circumstances. In iterative imaging and inversion methods, such as least-square reverse-time migration and full-waveform inversion, the computational cost is even larger because these processes require multiple rounds of wavefield modeling for gradient computation and optimal step size search to achieve inversion convergence and better spatial resolution. Under such circumstances, the computational cost associated with the wave equation modeling may become prohibitively high, even with modern parallel computing platforms.

One method to alleviate the computational burden is to solve the wave equation in a reduced model (e.g., [1,30,20,7,4,10,5,12]). Existing reduced-order methods show some advantages over conventional finite-difference or finite-element methods in terms of computational memory and time costs. Among these methods, the multiscale finite-element method (MsFEM) is a very reliable choice. It was originally developed for elliptic-type partial differential equations. The fundamental philosophy of MsFEM is to solve target partial differential equations on a coarse mesh instead of the conventional fine mesh, and accurate fine-scale solutions can be obtained by projecting the coarse-scale solution to the fine mesh using so-called multiscale basis functions. The multiscale basis functions contain the fine-scale heterogeneity information through some appropriate coupling mechanism. Specifically, the MsFEM [3,4] and its evolutionary version, the generalized multiscale finite-element method (GMsFEM) [10,5,12] have been proven to be efficient methods for acoustic-wave and elastic-wave equation modeling.

In general, the multiscale basis functions in the MsFEM are constructed by modifying the standard first-order finite-element shape functions. In models with large spatial grid size or highly complicated heterogeneities, wavefield solutions based on these first-order multiscale basis functions may not maintain sufficient accuracy over time evolution. The GMsFEM provides higher accuracy compared with the original MsFEM using multiple multiscale basis functions constructed from local eigenvalue problems [5,12]. However, an appropriate value of the number of multiscale basis functions for an arbitrary model is non-trivial to estimate. Occasionally, an adaptive strategy to assign different numbers of basis functions for different coarse elements in a coarse mesh is required, yet can complicate the computation.

To obtain high-order accurate solutions for the wave equation based on the multiscale finite-element theory, we develop a novel high-order MsFEM by constructing the multiscale basis functions using high-order spectral shape functions (i.e., Legendre polynomials). We call our new method the high-order multiscale finite-element method, or HMsFEM for short. In our HMsFEM, we employ the Gauss–Lobatto–Legendre (GLL) quadrature points (e.g., [18]) for integration computation in a coarse element. The most important feature of our HMsFEM is that, for each GLL points in a coarse element, we construct a multiscale basis function by solving a local elliptic problem (the static correspondence of the wave equation) with carefully designed boundary conditions and source terms. The multiscale shape functions are divided into two types depending on the GLL node's position in a coarse element: inside a coarse element or on the boundary of a coarse element. With these two types of shape functions, we construct the final high-order multiscale basis functions using a simple linear combination with combination coefficients determined from the orthogonality of the basis functions. As a result, the higher order multiscale basis functions can be viewed as a modification of the standard higher-order polynomials in FEM/SEM. Our HMsFEM can produce more accurate wavefield solutions compared with the MsFEM or GMsFEM, whose accuracy limit is usually the first-order SEM. With significantly reduced dimension of global matrices, our new HMsFEM can produce accurate wavefield solutions with much lower computational cost. With our newly developed HMsFEM, we aim to provide an efficient, affordable while sufficiently accurate tool for large-scale wave equation modeling in highly complicated media.

Our paper is organized as follows. In the Theory section, we develop the general formulation and discrete form of our HMsFEM for the wave equation. We then show how to construct the high-order spectral multiscale basis functions from local elliptic problems for the acoustic-wave equation. We also give a brief discussion of the implementation aspect of our new HMsFEM for the acoustic-wave equation. In the Numerical Results section, we use three numerical examples, including two 2D examples and one 3D example, to verify the superior computational efficiency and accuracy of our new HMsFEM. We conclude our method and numerical results in the Conclusions section.

2. Theory

We develop our multiscale method in the 3D scenario for generality. Formulations for the 2D case can be obtained with trivial modifications.

2.1. General formulation

We consider the acoustic wave equation in the second-order form

$$\frac{\partial^2 p}{\partial t^2} - \kappa \nabla \cdot (\rho^{-1} \nabla p) = f, \tag{1}$$

Download English Version:

https://daneshyari.com/en/article/6929046

Download Persian Version:

https://daneshyari.com/article/6929046

Daneshyari.com