Journal of Computational Physics 359 (2018) 106–136

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

Multi-resolution MPS method

Masayuki Tanaka [a,b,*], Rui Cardoso [a], Hamid Bahai [a]

[a] Brunel University London, Kingston Lane, Uxbridge, London, UB8 3PH, United Kingdom
[b] Corporate Manufacturing Engineering Center, Toshiba Co., Ltd., 33, Shinisogo-cho, Isogo-ku, Yokohama-shi, Kanagawa, 235-0017, Japan

ARTICLE INFO

Article history:
Received 27 July 2017
Received in revised form 27 November 2017
Accepted 29 December 2017
Available online 12 January 2018

Keywords:
Particle method
Least square MPS method
Multi-resolution
Incompressible fluid
Wall boundary
Pressure boundary

ABSTRACT

In this work, the Moving Particle Semi-implicit (MPS) method is enhanced for multi-resolution problems with different resolutions at different parts of the domain utilising a particle splitting algorithm for the finer resolution and a particle merging algorithm for the coarser resolution. The Least Square MPS (LSMPS) method is used for higher stability and accuracy. Novel boundary conditions are developed for the treatment of wall and pressure boundaries for the Multi-Resolution LSMPS method. A wall is represented by polygons for effective simulations of fluid flows with complex wall geometries and the pressure boundary condition allows arbitrary inflow and outflow, making the method easier to be used in flow simulations of channel flows. By conducting simulations of channel flows and free surface flows, the accuracy of the proposed method was verified.

1. Introduction

Full-Lagrangian particle methods have great advantages when compared with Eulerian methods due to the fact that the particle can move dynamically and there is no need to calculate the advection terms as in Eulerian methods. The absence of the advection terms makes it easier to simulate a free surface flow, that has been one of the most beneficial advantages of particle methods. Also, because connections to adjacent particles can be updated easily, the well-known excessive distortion problems common of a Lagrangian finite element procedure can be avoided. Originally, the Smoothed Particle Hydrodynamics (SPH) method, which is one of the most popular full-Lagrangian methods, was developed to simulate compressible fluid flows [32,7]. This method has been widely used to simulate not only compressible fluid but also incompressible fluid assuming that the fluid is weakly compressible. On the other hand, the Moving Particle Semi-implicit (MPS) method was developed to simulate incompressible fluid flow strictly within a full-Lagrangian context [23–25]. Although the basic concept of the MPS method is similar to the SPH method, the MPS method is unique because of the pressure term in the Navier–Stokes equation that is calculated implicitly by imposing the incompressible condition. A similar technique has been applied to SPH method and it is called the Incompressible SPH (ISPH) method [3].

The MPS method has been widely used to simulate incompressible fluid flows for several different applications such as the mixing of fluid as in the work of Matsunaga et al. [35], the numerical simulation of sloshing with large deforming free surfaces, Pan et al. [43], the three-dimensional numerical analysis of shipping water onto a moving ship, Shibata et al. [46], the numerical analysis of tsunami–structure interactions, Huang and Zhu [13], the numerical prediction of oil amount leaked from a damaged tank, Jeong et al. [15], the 3D simulation of the melting of metallic alloys, Mustari et al. [39], the numerical

* Corresponding author.
E-mail address: masayuki11.tanaka@toshiba.co.jp (M. Tanaka).

https://doi.org/10.1016/j.jcp.2017.12.042
0021-9991/

analysis of two-dimensional welding process, Saso et al. [45], the solidification of aluminium alloys in semi-solid forging processes, Regmi et al. [44], the simulation of Human swallowing, Kikuchi et al. [20] and the simulation of haemodynamics of small vessels, Gambaruto et al. [6].

However, the MPS method has a key fundamental instability problem that can deteriorate the accuracy of the solution considerably and even lead to a total failure of the simulation. It is well known that particle behaviour becomes unstable when attracting forces act between particles and this is called as tensile instability [51]. This problem is also very common in the SPH methods and there has been already many good research to handle tensile instability: Monaghan [37] introduced the artificial pressure technique to tackle tensile instability, Swegle et al. [51] explained the tensile instability by stating that it occurs when the direction of the force and the gradient of the kernel function were not consistent, Yang et al. [62] proposed a new kernel function whose second derivative was always positive and they showed that the tensile instability disappeared when this kernel function was used. However, Yang's kernel function cannot be applied to the MPS method because the second derivative of the kernel function does not appear in the MPS method.

The major cause for the source of tensile instability in the MPS method is the negative pressure around free surfaces of the flow and two techniques are commonly used to avoid the instability: altering the pressure gradient term and neglecting the negative pressure. In the former technique, the pressure gradient is evaluated by removing attracting forces. In the latter, the negative pressure is modified artificially to zero. These are not strict formulations but they are useful to improve stability. Using these techniques, the MPS method achieves great stability, however, they has been still suffering from inaccuracy.

Another potential source of instability is due to the inconsistency of the formulation of the MPS method. The MPS method is usually formulated by assuming that particles are arranged/distributed uniformly. The source of instability starts to be more evident when consistency is lost due to the non-uniform arrangement of particles as the simulation goes on. This inconsistency also leads to the loss of accuracy. The incompressible condition of the MPS method, which enforces the particle number density to be constant, is also one of the major sources for the loss of accuracy on the MPS method.

Many research was conducted in the past to improve the stability and accuracy of the MPS method. Tanaka and Masunaga [55] proposed a new formulation of the source term in the Poisson equation for pressure. Kondo and Koshizuka [22] proposed another formulation of the source term and this method was improved by Khayyer and Gotoh [17]. Shibata et al. [48] introduced the virtual particles method for the treatment of the negative pressure instability. Khayyer and Gotoh [16] altered the Laplacian model in 2D analysis and further extended it for 3D domains [18]. There has been some other relevant research for the improvement of the Laplacian model [41,53]. Tamai and Koshizuka [52] have proposed the Least Square MPS (LSMPS) method to overcome the inaccuracies caused by the non-uniform particle distribution and other similar methods were proposed for the SPH method [4,5].

Another limitation of the fully-Lagrangian particles method is computational cost. Generally, a particle method requires more computational resources when compared with an Eulerian method because of the extra computational effort required to handle particle interactions. However, some research was already conducted to tackle or minimise this issue by using multi-threading with multi-core CPU or GPU [11,65,38].

In Eulerian methods, changing the resolution of the discretisation for different parts of the domain is one of most distinguished characteristics of the method to reduce computational cost. In a Lagrangian description, the resolution, i.e. diameters of particles, is generally limited and non-uniform size diameters cannot be used. There were several attempts to change the spatial resolution in the SPH methods: Kitsionas and Whitworth [21] proposed the particle splitting method to increase the spatial resolution locally. In this method a particle is split into thirteen particles to obtain a spherically symmetric kernel function; Lastiwka et al. [27] proposed changing resolution by adding or removing particles. They applied a first-order differentiation scheme and showed that it was accurate even if particles had non-uniform spacing; Liu et al. [31] proposed the Adaptive SPH (ASPH) method, in which a support domain was extended to non-spherical regions such as an ellipsoidal shape region for example; Adams et al. [2] proposed a method to change the spatial resolution adaptively. All of the multi-resolution techniques developed for the SPH methods are based on formulations for a compressible fluid flow, therefore changing resolution is comparatively easy in this case.

A multi-resolution simulation for the MPS method is more difficult to be achieved because of the pressure term that has to be solved implicitly. However, there were some noteworthy developments for multi-resolution techniques for the MPS method such as the works of Shibata et al. [47] on the overlapping technique and Tang et al. [56] for the extension of Shibata's work to three dimensions. Notwithstanding the great contributions of Shibata and Tang for multi-resolution methods for the MPS method, there is still a major drawback which is the inability of the technique to allow for two-way interactions between low-resolution and high-resolution domains. Tanaka et al. [54] developed further a multi-resolution technique for the MPS method in two dimensions, however, the formulation was derived for the classical MPS method and thus it suffers from inaccuracy and stability issues. Tang et al. [57] extended this method for three dimensions, however, no splitting or merging algorithms were adopted and therefore the spatial resolution cannot be changed dynamically. The major conclusion to be taken is that a high accuracy and stability multi-resolution particle method for incompressible fluid flow still needs to be developed.

In this paper, the Multi-Resolution MPS method, which can change spatial resolution dynamically in incompressible fluid flow simulations is presented. New boundary conditions such as a wall and a pressure boundary conditions for multi-resolution simulations are also proposed.