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# A Coherent vorticity preserving eddy-viscosity correction for Large-Eddy Simulation

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ABSTRACT

This paper introduces a new approach to Large-Eddy Simulation (LES) where subgrid-scale (SGS) dissipation is applied proportionally to the degree of local spectral broadening, hence mitigated or deactivated in regions dominated by large-scale and/or laminar vortical motion. The proposed coherent-vorticity preserving (CvP) LES methodology is based on the evaluation of the ratio of the test-filtered to resolved (or grid-filtered) enstrophy, $\sigma$. Values of $\sigma$ close to 1 indicate low sub-test-filter turbulent activity, justifying local deactivation of the SGS dissipation. The intensity of the SGS dissipation is progressively increased for $\sigma < 1$ which corresponds to a small-scale spectral broadening. The SGS dissipation is then fully activated in developed turbulence characterized by $\sigma \leq \sigma_{eq}$, where the value $\sigma_{eq}$ is derived assuming a Kolmogorov spectrum. The proposed approach can be applied to any eddy-viscosity model, is algorithmically simple and computationally inexpensive. LES of Taylor–Green vortex breakdown demonstrates that the CvP methodology improves the performance of traditional, non-dynamic dissipative SGS models, capturing the peak of total turbulent kinetic energy dissipation during transition. Similar accuracy is obtained by adopting Germano's dynamic procedure albeit at more than twice the computational overhead. A CvP-LES of a pair of unstable periodic helical vortices is shown to predict accurately the experimentally observed growth rate using coarse resolutions. The ability of the CvP methodology to dynamically sort the coherent, large-scale motion from the smaller, broadband scales during transition is demonstrated via flow visualizations. LES of compressible channel are carried out and show a good match with a reference DNS.



## 1. Introduction

The Large-Eddy Simulation (LES) technique is a tool of prime importance as it enables an accurate numerical prediction of high-Reynolds number flows of practical relevance at accessible computational costs. In particular, the explicit representation of the spatial and temporal dynamics of vortices makes LES more accurate and versatile than Reynolds-Averaged Navier–Stokes (RANS) approaches, while its reduced requirements in terms of degrees of freedom compared to Direct Numerical Simulations (DNS) allows for tackling high-Reynolds number flows. The development of accurate and robust sub-grid scale (SGS) models extending the envelope of attainable Reynolds numbers, while containing the computational cost, is therefore still a warranted effort.

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One shortcoming of traditional LES modeling approaches is their tendency to introduce excessive SGS dissipation in transitional regions, impacting the evolution of the large coherent structures which may be on the verge of break-up. The Smagorinsky model [1], for example, attenuates velocity gradients at all scales of the flow, resulting in the undesired damping of coherent laminar vortices or transitional regions. Any accurate SGS model should therefore account for the energy transfers towards unresolved scales that are concentrated in the spectral neighborhood of the cutoff wavelength [2,3].

A number of approaches aim at correcting the overly dissipative nature of SGS models, and improving their spectral properties. The so-called Dynamic procedure adjusts automatically the subgrid model parameter, modulating the intensity of the subgrid dissipation for transitional or inhomogeneous flows [4–10]. Alternate multiscale approaches aim at applying a high-pass filter on the subgrid tensor to reduce the influence of the subgrid dissipation on the energy-containing large scales. The Variational Multiscale (VMS) approach introduced by Hughes et al. [11] and further developed in other works [12–19], aims at building a small-scale dissipative operator by performing an explicit scale-separation of the strain rate tensor. Among related approaches and variants, Stolz et al. [20] proposed to apply a high-pass filtered on the eddy viscosity, Vreman [21] applied a high-pass filter on the whole subgrid dissipative operator while Jeanmart and Winckelmans [22] applied the high-pass filter to the strain rate tensor. All the aforementioned approaches lead to significant improvement of the subgrid models accuracy. Other authors have proposed the use of turbulence sensors attempting to discriminate between laminar and turbulent regions [23–25]. Another approach termed Coherent Vortex Simulation (CVS) uses a wavelet-based decomposition to sort the coherent motion from the Gaussian component of the solution identified as small-scale noise [26–29], adopting a signal-processing and statistical approach. Alternative approaches aim at removing the subfilter energy by devising minimum-dissipation eddy-viscosity-based SGS models [30].

Purely numerical approaches have been proposed and rely on the use of regularization procedures such as explicit filtering or addition of artificial viscosity to counter the high-wave-number-energy accumulation occurring in low-dissipation numerical schemes [31–35].

Other approaches, termed Implicit LES (ILES), aim at tailoring the numerical dissipation naturally present in the adopted numerical discretization, to mimic the sub-grid dissipation deriving from physical SGS models [36–39]. Although resulting in almost no computational overhead, as no explicit execution of subgrid models is needed, such techniques may still introduce excessive dissipation [40,41], with less flexibility than inherently non-dissipative numerical schemes equipped with subgrid models, which can be deactivated in dynamically selected regions of the flow. The latter will be the approach followed in this study.

In fact, accurate results have been obtained in the past by coupling high-order finite difference schemes and Dynamic models [42–44]. Classic Dynamic modeling approaches, however, increase cost, memory requirements and complexity of the implementation, associated with the test-filtering of tensors and averaging of the dynamic parameter along directions of statistical homogeneity [4] or flow path trajectories [6] to obtain stable computations.

In this paper, we present a new strategy for quantifying the local degree of spectral broadening in the flow with a simple and computationally inexpensive turbulence sensor identifying regions of developed, locally high Reynolds number turbulence requiring SGS dissipation. This approach can be seen as a new dynamic approach, blending physics-based SGS modeling with a new scale-selective sensor based on the evaluation of the sub-test-filter enstrophy content. By test-filtering vorticity rather than velocity, a greater sensitivity to the emergence of small-scales in the flow is achieved. Indeed, while the bulk of the energy in the flow is carried by the large-scale motion, the small scales are characterized by high levels of enstrophy. Hence, defining a scale separation of enstrophy is a natural choice for the development of a sensor detecting small-scale dynamics. By evaluating the ratio of grid- and test-filtered enstrophy, it is possible to quantify the relative small-scale energy content of the flow to then mitigate SGS dissipation in non-turbulent, large-scale narrowband vorticity dominated regions where coherent or large-scale vortices are likely to be found. The sensor is hence able to discriminate between gradients due to small scale vorticity (most likely broadband turbulence) and large scale structures, which are not governed by inertial subrange transfer energy dynamics assumed by most SGS models. The subgrid dissipation will therefore be activated only in regions where under-resolved, small-scale turbulence is prominent. Due to this property we refer to the proposed method as coherent-vorticity preserving.

The present approach has a number of advantages: (1) the sensor improves the accuracy of any existing SGS model in transitional flows; (2) the sensor is based on local and instantaneous flow values, allowing for a dynamic adjustment of the SGS dissipation; (3) the CvP technique is computationally inexpensive, only requiring one additional test-filtering operation on the enstrophy field; (4) algorithmically simple, not requiring any spectral decomposition of the flow and easily extendable to unstructured meshes.

As advanced subgrid modeling approaches such as the Dynamic model or the VMS approaches provide as well a good accuracy for transitional and complex flows, the CvP methodology is expected to be computationally less intensive. Indeed, the VMS requires the scale separation of all velocity gradients in order to build the small-scale strain-rate tensor and the Dynamic approach requires the test filtering of numerous quantities for computing the dynamic parameter. The CvP-LES approach requires the filtering of only one quantity, the enstrophy, yielding a minimal computational overhead.

The outline of the paper is as follows. First, the LES formalism and numerical methods are detailed in section 2. The CvP technique is derived in section 3. The various subgrid-scale models used in the study are reported in section 3.2. Section 4 features a sensitivity study of the CvP technique to the grid resolution and test filter width in LES of transitional Taylor–Green vortex. The sensor is coupled to various traditional dissipative SGS models, and compared to the classical