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# Hybrid-dimensional modelling of two-phase flow through fractured porous media with enhanced matrix fracture transmission conditions

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In this work, we extend, to two-phase flow, the single-phase Darcy flow model proposed in [26], [12] in which the $(d-1)$-dimensional flow in the fractures is coupled with the $d$-dimensional flow in the matrix. Three types of so called hybrid-dimensional two-phase Darcy flow models are proposed. They all account for fractures acting either as drains or as barriers, since they allow pressure jumps at the matrix–fracture interfaces. The models also permit to treat gravity dominated flow as well as discontinuous capillary pressure at the material interfaces. The three models differ by their transmission conditions at matrix fracture interfaces: while the first model accounts for the nonlinear two-phase Darcy flux conservations, the second and third ones are based on the linear single phase Darcy flux conservations combined with different approximations of the mobilities. We adapt the Vertex Approximate Gradient (VAG) scheme to this problem, in order to account for anisotropy and heterogeneity aspects as well as for applicability on general meshes. Several test cases are presented to compare our hybrid-dimensional models to the generic equi-dimensional model, in which fractures have the same dimension as the matrix, leading to deep insight about the quality of the proposed reduced models.



## 1. Introduction

This work is concerned with the modelling and the discretization of two-phase Darcy flows in fractured porous media, for which the fractures are represented as interfaces of codimension one. In this framework, the $(d-1)$-dimensional flow in the fractures is coupled with the $d$-dimensional flow in the matrix leading to the so called, hybrid-dimensional Darcy flow models. These models are derived from the so called equi-dimensional model, where fractures are represented as geological structures of equal dimension as the matrix, by averaging fracture quantities over the fracture width. We consider the case for which the pressure can be discontinuous at the matrix–fracture $(mf)$ interfaces in order to account for fractures acting either as drains or as barriers as described in [19,26,6,12], contrary to the continuous pressure model described in [3,11] developed for conductive fractures.

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Our objective is to compare different models extending hybrid-dimensional discontinuous pressure models to two-phase Darcy flows. The comparison between models uses the equi-dimensional model to provide a reference solution. A rather simple 2D setting is considered to be able to compute this reference solution and make the comparison. The targeted hybrid-dimensional model should be able to account accurately for gravity effects inside the fractures, for discontinuous capillary pressure curves at $mf$ interfaces and for both drains and barriers. Most existing hybrid-dimensional two-phase Darcy flow models are based on the assumption of continuous phase pressures at the $mf$ interfaces assuming fractures acting as drains [8,31,30,24,9,10]. Another existing approach, accounting for both drains or barriers, amounts to eliminate the interfacial phase pressures using the linear single phase Darcy flux conservation equation at the $mf$ interfaces for each phase. It is usually combined with Two Point [25] or MultiPoint [35,33,1] cell centred finite volume schemes for which the interfacial unknowns can be easily eliminated when building the single phase Darcy flux transmissivities. It is important to notice that, in the case of fractures acting as drains, this approach provides basically the same solutions as the ones obtained by the continuous pressure model. These hybrid-dimensional models are here compared with a new hybrid-dimensional model which

- accounts for the $mf$ interfacial phase pressures in addition to the phase pressures averaged along the width of the fracture,
- models the two-phase Darcy normal fluxes in the fracture at each $mf$ interface using a Two Point Flux Approximation with phase based upwinding, gravity term and saturation jump,
- writes the nonlinear two-phase Darcy flux conservation equations at the $mf$ interface.

To our knowledge, this type of hybrid-dimensional two-phase Darcy flow model has not been studied before, except in [27,2] where the authors use a global pressure formulation. In order to account more efficiently for discontinuous capillary pressures, our model uses the phase pressures as primary unknowns. This formulation is based on the inverse of the monotone graph extension of the capillary pressure curves and can be easily extended to general capillary pressure curves including vanishing capillary pressures in the fracture using a switch of variable formulation as described in [10] in the case of continuous pressure models. Our coupling conditions at the $mf$ interfaces also differ from the ones presented in [27,2] in the sense that they incorporate an upwinding between the matrix and fracture mobilities and do not neglect the gravitational force. This upwinding is crucial in order to transport fluid from the matrix to the fractures and the gravitational force in the width of the fracture cannot be neglected for gravity dominated flows independently on the fracture width.

An important novelty of the proposed hybrid-dimensional model is that the saturations at the $mf$ interfaces are explicitly calculated, in addition to the mean saturations along the width of the fractures. All of the aforementioned models [8,31,30,24,9,10,25] lack in this supplementary information. They either have only the mean physical unknowns along the width of the fractures, which is the case for the hybrid-dimensional continuous pressure models, or they eliminate the interfacial pressures using the linear single phase flux conservation equations. Note again that this linear elimination provides basically the same results as continuous pressure models in the case of fractures acting as drains. The importance of preserving both the $mf$ interfacial and mean saturations becomes obvious in the test case section of this work: the influence on the solution of capillary or gravitational forces in normal direction within the fractures is far from being negligible, in general. The supplementary unknowns at the $mf$ interfaces enable the method presented in this paper to capture these effects. This is of course a very different situation than what occurs for a single phase flow possibly coupled with a linear transport tracer equation for which the pressure (and possibly tracer) continuity assumption at the $mf$ interfaces provides very accurate results in the case of fractures acting as drains.

The discretization of hybrid-dimensional Darcy flow models has been the object of several works. For a review of existing methods, we refer to [32,34]. For single-phase Darcy flow, a cell-centered Finite Volume scheme using a Two Point Flux Approximation (TPFA) is proposed in [25,19,6] assuming the orthogonality of the mesh and isotropic permeability fields. Cell-centred Finite Volume schemes can be extended to general meshes and anisotropic permeability fields using MultiPoint Flux Approximations (MPFA) following [35,33,1]. Nevertheless, MPFA schemes can lack robustness on distorted meshes and for large anisotropies due to the non symmetry of the discretization. They are also very expensive compared to nodal discretizations on tetrahedral meshes. In [26], a Mixed Finite Element (MFE) method is proposed. More recently the Hybrid Finite Volume (HFV) scheme, introduced in [15], has been extended in [18] for the geometrically non-conforming discretization of two reduced fault models. Also a Mimetic Finite Difference (MFD) scheme is used in [7] in the matrix domain coupled with a TPFA scheme in the fracture network. Discretizations of the related reduced model [3] assuming a continuous pressure at the $mf$ interfaces have been proposed in [3] using a MFE method and in [11] using the HFV scheme and an extension of the Vertex Approximate Gradient (VAG) scheme introduced in [14]. Finally, the VAG and HFV schemes have been extended to the single-phase hybrid-dimensional discontinuous pressure model in [12]. Let us also mention two classes of so-called geometrically non-conforming discretizations, that handle non-matching fracture and matrix meshes: the Extended Finite Element Method [5,20,21] and the Embedded Discrete Fracture Method [29,23].

For two-phase Darcy flow, the TPFA discretization is used in [25] with elimination of the $mf$ interface pressures when computing the Darcy flux transmissivities. In [24], the two-phase flow equations are solved in an IMPES framework, using a Mixed Hybrid Finite Element (MHFE) discretization for the pressure equation and a Discontinuous Galerkin discretization of the saturation equation. Either a zero flux or the pressure continuity are assumed at $mf$ interfaces. The paper also contains a review on the most common numerical approaches, when dealing with discrete fractures. The Hybrid Finite Volume