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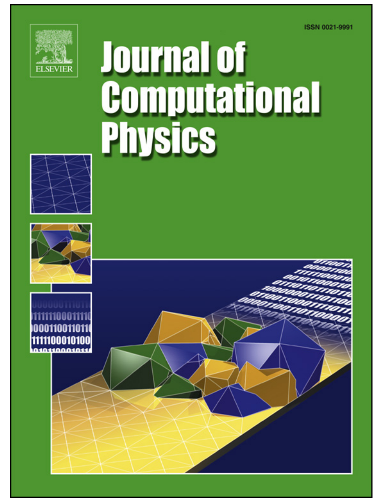

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Hidden Physics Models: Machine Learning of Nonlinear Partial Differential Equations

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Abstract

While there is currently a lot of enthusiasm about "big data", useful data is usually "small" and expensive to acquire. In this paper, we present a new paradigm of learning partial differential equations from *small* data. In particular, we introduce *hidden physics models*, which are essentially data-efficient learning machines capable of leveraging the underlying laws of physics, expressed by time dependent and nonlinear partial differential equations, to extract patterns from high-dimensional data generated from experiments. The proposed methodology may be applied to the problem of learning, system identification, or data-driven discovery of partial differential equations. Our framework relies on Gaussian processes, a powerful tool for probabilistic inference over functions, that enables us to strike a balance between model complexity and data fitting. The effectiveness of the proposed approach is demonstrated through a variety of canonical problems, spanning a number of scientific domains, including the Navier-Stokes, Schrödinger, Kuramoto-Sivashinsky, and time dependent linear fractional equations. The methodology provides a promising new direction for harnessing the long-standing developments of classical methods in applied mathematics and mathematical physics to design learning machines with the ability to operate in complex domains without requiring large quantities of data.

Keywords: probabilistic machine learning, system identification, Bayesian modeling, uncertainty quantification, fractional equations, small data