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Moving mesh finite element simulation for phase-field modeling of brittle fracture and convergence of Newton's iteration

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ABSTRACT

A moving mesh finite element method is studied for the numerical solution of a phase-field model for brittle fracture. The moving mesh partial differential equation approach is employed to dynamically track crack propagation. Meanwhile, the decomposition of the strain tensor into tensile and compressive components is essential for the success of the phase-field modeling of brittle fracture but results in a non-smooth elastic energy and stronger nonlinearity in the governing equation. This makes the governing equation much more difficult to solve and, in particular, Newton's iteration often fails to converge. Three regularization methods are proposed to smooth out the decomposition of the strain tensor. Numerical examples of fracture propagation under quasi-static load demonstrate that all of the methods can effectively improve the convergence of Newton's iteration for relatively small values of the regularization parameter but without compromising the accuracy of the numerical solution. They also show that the moving mesh finite element method is able to adaptively concentrate the mesh elements around propagating cracks and handle multiple and complex crack systems.

1. Introduction

Brittle fracture is the fracture of a metallic object or other elastic material where plastic deformation is strongly limited. It usually occurs very rapidly and can be catastrophic in engineering practice; e.g., see Pokluda and Šandera [48]. Understanding the initiation and propagation of brittle fracture and preventing fracture failure are vital to the engineering design, where numerical simulation of fracture processes has become a powerful tool. Computational approaches for studying brittle fracture can be roughly categorized into two groups, discrete crack models and smeared crack models. In the former group, discontinuous fields are introduced into the numerical model and cracks are described as moving boundaries. One major challenge for those models is to track moving boundaries. A commonly used strategy is to change the mesh geometry by introducing new boundaries at each time step together with adaptive remeshing; e.g., see [6,12,30,47]. The mesh regenerating and boundary updating not only increase computational cost but also further complicate the implementation of boundary conditions. In order to avoid complex remeshing, Moës et al. [43,44] propose the extended finite element method, which enriches the finite element spaces with discontinuous fields based on the partition-of-unity concept and allows the

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propagation of cracks along element interfaces. A drawback for the method is that it requires explicit description of crack patterns and thus has difficulty in dealing with complex cracks and unforeseen patterns of crack propagation.

In the second group of computational approaches, smeared crack models approximate cracks with continuous fields and do not rely on explicit description of cracks. The phase-field model based on the variational approach proposed by Francfort and Marigo [17] is a commonly used type of smeared crack model. In the phase-field modeling, a phase-field variable d, which depends on a parameter l describing the actual width of the smeared cracks, is introduced to indicate where the material is damaged. One of the major advantages of this model is that the initiation and propagation of cracks are completely determined by a coupled system of partial differential equations based on the energy functional. Another advantage is that the generation and propagation of fracture networks do not require explicitly tracking fracture interfaces. The phase-field modeling is used in this work.

The phase-field modeling has been successfully applied in many other fields including image segmentation [2], dendritic crystal growth [31,56], and multiple-fluid hydrodynamics [34,50,51,59]. Since its first application in brittle fracture simulation by Bourdin et al. [13], significant progress has been made in this area; e.g., see [3,9–11,32,35,38,40,42,45,53]. However, there still exist challenges. In particular, the strain tensor has to be decomposed along eigen-directions into tensile and compressive components in the presence of cracks, with only the former component contributing to generation and propagation of cracks. This decomposition of the strain tensor is introduced by Miehe et al. [40] to account for the reduction of the stiffness of the elastic solid by cracks and to rule out unrealistic branching. Unfortunately, it also makes the elastic energy non-smooth and increases the nonlinearity of the governing equation. As a consequence, the Jacobian matrix of the governing equation may not exist at places and Newton's iteration can often fail to converge [35,45].

The phase-field modeling is governed by a coupled system for the phase-field variable d and the displacement u which can be solved in the monolithic or staggered approach. In the monolithic approach (e.g., see [19,57,58]), the system is solved simultaneously for both d and u, and it is often challenging to obtain a convergent solution due to the non-convexity nature of the energy functional. A damped Newton method with linear search [19] and an error-oriented Newton method [57] have been developed to overcome convergence issues while a modified Newton scheme with Jacobian modification has been proposed by Wick [58]. The staggered approach solves the coupled system sequentially for d and u and has been used by a number of researchers; e.g., see [1,3,18,40,42]. By fixing one variable such as the phase-field d, the underlying problem becomes convex in the other unknown variable u. Since d and u are not coupled, the procedure becomes simpler and more robust. The main disadvantage of this approach is that at a loading step many staggered iterations are required to reach convergence, which can be costly. The effects of the number of staggered iterations in relation to the size of the loading increment has been studied by Ambati et al. [1]. Their results show that an insufficient number of d–u iterations can lead to inaccurate results when large loading increments are used. However, the number of staggered iterations usually has relatively little performance impact on the shape of the load-displacement curves when loading increments are sufficiently small. This implies that the staggered approach without iteration can be used as long as small loading increments are taken. Since our focus in this work is on mesh adaptation and convergence of Newton's iteration, we use the staggered approach without iteration for quasi-static brittle fracture problems with small loading increments.

It is worth mentioning that for the staggered approach with/without iteration, the equation for u remains highly nonlinear due to the decomposition of the strain tensor, which can make Newton's iteration fail or be slow to converge; see [1] or the numerical results in Section 4. On the other hand, like implicit schemes versus explicit schemes for ordinary differential equations, the staggered approach with iteration or the monolithic approach can be more robust than the staggered approach without iteration which is used in the current work. Comparison of their performances with mesh adaptation and regularization of the decomposition of the strain tensor (see discussion below) can be an interesting topic for future investigations.

The model parameter that describes the width of smeared cracks is needed to be small for the phase-field model to be a reasonably accurate approximation of the original problem. This in turn requires that the mesh elements be small at least in the crack regions, meaning that mesh adaptation is necessary to improve the computational accuracy and efficiency. The mesh adaptation should be dynamical too since cracks can propagate under continuous load.

Moving mesh methods are well suited for the numerical simulation of the phase-field modeling of brittle fracture. Although they have been successfully applied to phase-field models for other applications, e.g., see [16,36,49,54,60,61], they have not been employed for brittle fracture simulation, which has distinguished challenges associated with the above mentioned decomposition of the strain tensor. As a matter of fact, mesh adaptation has rarely been employed in brittle fracture simulation so far and there are only a few published studies on the topic. Noticeably, Heister et al. [20] develop a predictor–corrector local mesh adaptivity scheme that allows the mesh to refine around cracks. Artina et al. [5] present an a posteriori error estimator for anisotropic mesh adaptation that generates thin, anisotropic elements around cracks and isotropic elements away from the cracks, but they use an early model that does not decompose the strain tensor and therefore does not distinguish between fracture caused by tension and compression.

The objective of this paper is twofold. The first is to study the MMPDE (moving mesh partial differential equation) moving mesh method [14,26–28] for the phase-field modeling of brittle fracture. The MMPDE method is a type of dynamic mesh adaptation method specially designed for time dependent problems. It employs a mesh PDE to move the mesh continuously in time to follow and adapt to evolving structures in the solution. A new formulation of the MMPDE method was developed recently in [24], which provides a simple, compact analytical formula for the nodal mesh velocities (cf. (23) below), and this makes its implementation relatively easy. It is also shown in [25] that the mesh governed by the underlying mesh equation