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Lubricated immersed boundary method in two dimensions

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ABSTRACT

Many biological examples of fluid–structure interaction, including the transit of red blood cells through the narrow slits in the spleen and the intracellular trafficking of vesicles into dendritic spines, involve the near-contact of elastic structures separated by thin layers of fluid. Motivated by such problems, we introduce an immersed boundary method that uses elements of lubrication theory to resolve thin fluid layers between immersed boundaries. We demonstrate 2nd-order accurate convergence for simple two-dimensional flows with known exact solutions to showcase the increased accuracy of this method compared to the standard immersed boundary method. Motivated by the phenomenon of wall-induced migration, we apply the lubricated immersed boundary method to simulate an elastic vesicle near a wall in shear flow. We also simulate the dynamics of a vesicle traveling through a narrow channel and observe the ability of the lubricated method to capture the vesicle motion on relatively coarse fluid grids.

1. Introduction

The immersed boundary method is a widely-used numerical method for fluid–structure interaction that has been applied to problems including blood clotting [1], osmotic swelling due to thermal fluctuations [2], sperm locomotion through viscoelastic fluids [3], and insect flight [4]. Several extensions of the immersed boundary method have been developed to incorporate realistic structural properties such as added mass [5], permeability [6,7], intrinsic twist [8], nonlinear constitutive laws [9], and variable viscosity and density [10,11]. The chief idea of the immersed boundary method is to use an Eulerian description of the fluid and a Lagrangian description of the structure, while coupling these descriptions through integral operators involving delta functions. In the continuum formulation, these are the usual Dirac delta functions, and much of the efficiency and accuracy of the immersed boundary method depends on how these Dirac delta functions are regularized in the discrete setting of the numerical method [12,13].

One of the features of the immersed boundary method is that it is formulated in a manner that avoids the contact problem; since all structures move in the same global velocity field, they cannot in principle cross themselves or one another [14]. However, advecting all structures in the same regularized velocity field presents challenges when multiple immersed boundaries are in near contact, which occurs frequently in biological phenomena such as red blood cell motion through the microcirculation [15] and membrane receptor trafficking in dendrites [16,17]. In simulations that use uniform fluid grids, the flow in the thin fluid layer between immersed boundaries may not be sufficiently resolved, resulting in boundaries that are effectively stuck together. This is sometimes dealt with by the addition of repulsive forces, as done

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by Krishnan et al. to prevent "unphysical overlap" of rigid particles [18], and by Lim et al. in their simulations of flagellar bundling in *E. coli* to "prevent filaments from crossing each other" [19]. Instead of applying additional forces, we wish to overcome this difficulty by using an asymptotic limit of the governing equations.

The immersed boundary method takes the viewpoint that the entire domain, including those regions containing immersed elastic structures, is filled with fluid having a smoothly varying velocity field. The smoothness of the velocity field implies that there are no large jumps in the velocities of structures that are nearby relative to the grid spacing Δx, and consequently the immersed boundary method can yield inaccurate results for problems in which these large velocity gradients do arise, such as when two structures that are separated by a thin fluid layer slide relative to one another.

Given the difficulties observed above, we propose a modification to the immersed boundary method that makes use of lubrication theory to resolve the flow through thin fluid layers. Unlike the standard immersed boundary method described thus far, which becomes less accurate as two boundaries approach each other, the lubrication approximation actually *improves* as the gap size decreases. The grid spacing Δx provides a natural cutoff at which to apply the lubrication approximation. In the proposed method, the lubrication approximation is automatically turned off when the gap size becomes larger than a few times the grid spacing, as described in detail later on.

We illustrate this method by applying it to two test problems of increasing complexity for which the lubricated immersed boundary method has significantly smaller errors than the standard method. The primary additional cost of the lubricated immersed boundary method involves the construction of a height function that gives the distance between the immersed boundaries. We describe a method to compute this height function accurately and efficiently. Finally, we apply the lubricated immersed boundary method to study the motion of elastic vesicles near walls and observe the significantly improved accuracy obtained in simulations of wall-induced migration and channel flow on coarse fluid grids.

Our work builds on several previous efforts to merge direct numerical simulation with lubrication theory. In the context of particle suspensions, Nguyen and Ladd and Janoschek et al. worked out lubrication corrections to lattice-Boltzmann simulations of particle suspensions [20,21], while Seto et al. performed simulations of hard spheres with regularized lubrication forces [22]. Thomas et al. performed a multiscale computation of a fluid drop interacting with a wall in which lubrication theory was used to replace the no-slip boundary condition by a specified wall-shear stress [23]. Our approach is most closely related to this last work, since we also combine a grid-based fluid solver with analytical results from lubrication theory to model subgrid effects. There are common features in the mathematical formulations; we also use a height function to describe the size of the lubricating layer between boundaries, and we also relate the flow in the gap to the stress on the boundaries. However, whereas Thomas et al. solve an evolution equation for the height and flux through the gap, the solution to which gives a boundary condition for the fluid, our boundaries are defined in a Lagrangian manner, with the height function constructed based on an explicit representation of the boundaries in terms of piecewise polynomial curves. In our approach, we use lubrication theory to modify the advection velocity. Further, whereas Thomas et al. considered a multiphase fluid in which a droplet of fluid having no surface elasticity and ten times greater density and viscosity than the ambient fluid falls down a slope, in the present study we restrict attention to elastic structures immersed in fluids with constant density and viscosity.

We note that an alternative approach for ensuring that the fluid dynamics is sufficiently resolved throughout the domain is to use adaptive mesh refinement (AMR). The idea of AMR is to use a hierarchy of fluid grids with different levels of refinement, with finer grids placed in the domain as needed to obtain a desired accuracy [24]. This allows for highly-resolved computations at drastically reduced computational cost compared to uniform grids. However, implementing AMR requires additional data structures to keep track of the grid hierarchy, and typical implementations of AMR require the user to specify a finest grid level so that issues with near-contact are not definitively resolved. This finest grid size can also lead to severe timestep restrictions for numerical stability. The advantage of using a subgrid model, as we do here through lubrication theory, is that the asymptotic results hold for infinitesimally small gaps, and does not introduce additional timestep constraints. Further, relatively little additional machinery is required beyond what is already used for uniform grid simulations; as we will discuss later on, the primary additional cost of the lubricated immersed boundary method is the construction of the height function that encodes the distance between the immersed boundaries.

2. Formulation

First, we review the standard immersed boundary method, so that the new features of the lubricated method will be clear as they are introduced later on.

2.1. Standard immersed boundary method

Consider a domain Ω filled with incompressible fluid of density ρ and viscosity μ, having velocity $\mathbf{u}(\mathbf{x}, t)$ and pressure $p(\mathbf{x}, t)$ defined in terms of cartesian (a.k.a. Eulerian) coordinates $\mathbf{x}$. For many relevant problems, the nonlinear term in the Navier–Stokes equations may be neglected, and we will restrict attention to this case in which the fluid is described by the unsteady Stokes equations: