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Spectral treatment of gyrokinetic shear flow

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ABSTRACT

Sheared $\mathbf{E} \times \mathbf{B}$ flow in a tokamak, driven by external torque from neutral beam injection, is known to have an important stabilizing effect on drift-wave turbulence. In gyrokinetic codes, flow shear can be implemented directly on a radial mesh with nonperiodic boundary conditions. The mesh-based implementation is straightforward, but carries the possibility of spurious effects related to simulation boundaries. Alternatively, flow shear has been implemented in spectral solvers using a wavenumber shift method. Although the spectral representation has numerous computational benefits, the wavenumber shift method for treating flow shear is of questionable accuracy. Efforts to compare mesh-based solutions with spectral ones have met with limited success. In particular, significant differences in the critical shear required to stabilize turbulence are sometimes observed. We outline a new approach to treat flow shear spectrally. The method is simple to implement, matches the nonperiodic results more closely, and predicts a critical shear that is less sensitive to radial wavenumber resolution.

1. Introduction and background

1.1. History

External torque from neutral beam injection in tokamak plasmas can produce toroidal rotation velocities comparable to the ion sound speed. The radial variation (shear) in the corresponding $\mathbf{E} \times \mathbf{B}$ velocity is known to have an important stabilizing effect on drift-wave turbulence, thereby improving tokamak confinement. The importance of including this effect in nonlinear simulations of tokamak turbulence has long been known [1–5]. However, the way the effect is implemented depends on the simulation type; this can be either a radially-nonperiodic annulus [6], or a radially-periodic flux-tube [7]. In the former case, the shearing can be calculated simply and directly on a radial mesh with nonperiodic boundary conditions, such that calculated fluxes converge as the mesh is refined. In the latter case, however, the situation is more complicated. To understand the nature of this complication, note first that spectral flux-tube gyrokinetic solvers are computationally very efficient for certain problems since they work directly in wavenumber space and employ periodic radial and toroidal boundary conditions. In particular, simulations that require simultaneous resolution of electron-scale and ion-scale turbulence [8,9] benefit greatly from spectral algorithms. Unfortunately, periodic radial boundary conditions are fundamentally incompatible with flow shear. The standard method (herein called the *wavenumber-shift* method, and elsewhere called the *wavenumber-remapping* method [10]) gets around this problem by making use of the fact that toroidal harmonics return to radial periodicity at integer multiples of a critical time [11,12]. For a fixed radial domain size, however, the discretization

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error is fixed and cannot be reduced by resolving higher wavenumbers. Thus, proving convergence via mesh refinement is not possible. Efforts to compare results from real-space and spectral implementations have been minimal, and in some cases significant differences in predictions of the critical shearing rate to stabilize turbulence have been observed [10,13]. In this work, we outline a new approach to the spectral shearing method that is very simple to implement, matches the global method more closely for a standard nonlinear test case, and predicts a critical shear that is relatively insensitive to wavenumber resolution.

1.2. Theoretical considerations

In the recursive formulation of nonlinear electromagnetic gyrokinetic theory [14,15], fluctuations are represented in eikonal (ballooning) form as

$$h_a(\mathbf{R}) = \sum_{\mathbf{k}_\perp} e^{i\mathrm{S}_\mathbf{k}(\mathbf{R},t)} h_{a,\mathbf{k}_\perp} \; , \tag{1}$$

where $\mathbf{k}_\perp \doteq \nabla_\perp \mathrm{S}_\mathbf{k}$ is the perpendicular wavenumber, $\mathbf{R}$ is the location of a gyrocenter, and a is the species index. When the plasma has a flow velocity $\mathbf{V}_0$, the eikonal becomes time dependent [16,2,14,15], such that

$$\frac{\partial \mathrm{S}_\mathbf{k}}{\partial t} = -\mathbf{k}_\perp \cdot \mathbf{V}_0 \; . \tag{2}$$

In a tokamak, any mean flow on the order of a thermal ion velocity must be purely toroidal and of the form $\mathbf{V}_0 = R^2\omega_0(\psi)\nabla\varphi$ [17]. In terms of the nonrotating part of the eikonal, $\mathrm{S}_\mathbf{k}(0)$, we have

$$\mathrm{S}_\mathbf{k}(t) = \mathrm{S}_\mathbf{k}(0) + n\omega_0 t \; , \tag{3}$$

$$\mathbf{k}_\perp(t) = \mathbf{k}_\perp(0) + n\nabla r \frac{\partial \omega_0}{\partial r} t \; , \tag{4}$$

where $n \doteq -R^2\mathbf{k}_\perp \cdot \nabla\varphi$, and r is the *midplane minor radius* (the half-width of the flux surface at the elevation of the centroid [18]). In this paper, we use the non-orthogonal field-aligned coordinate system (ψ, θ, α) together with the Clebsch representation for the magnetic field [19], $\mathbf{B} = \nabla\alpha \times \nabla\psi$. Here, $\alpha \doteq \varphi + \nu(\psi, \theta)$, where φ is the toroidal angle, ψ is the poloidal flux divided by 2π, and θ is the poloidal angle. The safety factor, q, is defined as

$$q(\psi) \doteq \frac{1}{2\pi} \int_0^{2\pi} \frac{\mathbf{B}\cdot\nabla\varphi}{\mathbf{B}\cdot\nabla\theta} \, d\theta = \frac{1}{2\pi} \int_0^{2\pi} \left(-\frac{\partial \nu}{\partial \theta} \right) d\theta = \frac{\nu(\psi, 0) - \nu(\psi, 2\pi)}{2\pi} \; . \tag{5}$$

With these results, the eikonal can be put into a more useful and intuitive form by introducing the *poloidal wavenumber* $k_\theta = nq/r$ and the *Waltz shearing rate* [1],

$$\gamma_E \doteq -\frac{r}{q} \frac{d\omega_0}{dr} \; , \tag{6}$$

to yield

$$\mathrm{S}_\mathbf{k}(t) = \mathrm{S}_\mathbf{k}(0) + n\omega_0(r_0)t - (r - r_0)k_\theta \gamma_E t \; , \tag{7}$$

$$\mathbf{k}_\perp(t) = \mathbf{k}_\perp(0) - \nabla r k_\theta \gamma_E t \; . \tag{8}$$

To arrive at this result, we have expanded the rotation frequency ω_0 about the center of the simulation domain $r = r_0$:

$$\omega_0 \sim \omega_0(r_0) + \left[\frac{d\omega_0}{dr} \right]_{r=r_0} (r - r_0) \; . \tag{9}$$

In what follows we will neglect the $\omega_0(r_0)$ term in Eq. (7) since it represents a simple Doppler shift in the frequency.

2. Periodic and nonperiodic series

For numerical simulation, fluctuations for plasma species a can in principle be decomposed using an explicit double Fourier series with *time-dependent* eikonal

$$h_a(x, \alpha, t) = f_{0a} \sum_{n=-N}^{N} \sum_{p=-M}^{M} e^{ipx} e^{-ix\gamma_s t} e^{-in\alpha} \widehat{h}_a(n, p, t) \; . \tag{10}$$