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A point-value enhanced finite volume method based on approximate delta functions

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ABSTRACT

We revisit the concept of an approximate delta function (ADF), introduced by Huynh (2011) [1], in the form of a finite-order polynomial that holds identical integral properties to the Dirac delta function when used in conjunction with a finite-order polynomial integrand over a finite domain. We show that the use of generic ADF polynomials can be effective at recovering and generalizing several high-order methods, including Taylor-based and nodal-based Discontinuous Galerkin methods, as well as the Correction Procedure via Reconstruction. Based on the ADF concept, we then proceed to formulate a Point-value enhanced Finite Volume (PFV) method, which stores and updates the cell-averaged values inside each element as well as the unknown quantities and, if needed, their derivatives on nodal points. The sharing of nodal information with surrounding elements saves the number of degrees of freedom compared to other compact methods at the same order. To ensure conservation, cell-averaged values are updated using an identical approach to that adopted in the finite volume method. Here, the updating of nodal values and their derivatives is achieved through an ADF concept that leverages all of the elements within the domain of integration that share the same nodal point. The resulting scheme is shown to be very stable at successively increasing orders. Both accuracy and stability of the PFV method are verified using a Fourier analysis and through applications to the linear wave and nonlinear Burgers' equations in one-dimensional space.

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1. Introduction

The Dirac delta function represents a well-defined distribution that extends over a line of real numbers while possessing the unique property of vanishing everywhere except at the origin. Nonetheless, it still produces a unit value when integrated over the entire real line. Moreover, one of its most distinguishing properties stands, perhaps, in its ability to reproduce the values and derivatives of any function in integral form. In this paper, we show that the integral properties of the delta function may be useful in a number of computational settings as an alternative vehicle for evaluating functional values and derivatives over a finite domain. In numerical computations, however, the theoretical delta function suffers from singularities because of its sudden vanishing and infinite distribution. In 2011 and 2014, Huynh [1,2] introduced a very important concept, namely, that of an approximate delta function (ADF), which serves well to overcome these limitations. Accordingly, the ADF is defined as a finite-order polynomial that is capable of preserving the integral properties of the exact delta function in the evaluation of finite-order polynomials over finite domains. In this study, we extend the ADF concept

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by allowing the ADF polynomial to contain arbitrary coefficients and by defining ADF derivative weight functions that can be very effective in the development of a high-order numerical framework for solving partial differential equations.

It is well known that, in the field of computational fluid dynamics, low-order methods are often selected because of their simplicity and robustness, factors that jointly justify their recurrent use in engineering practice. Using similar CPU resources, however, high-order methods can provide more accurate solutions, albeit at the cost of increased complexity and reduced robustness. For this reason, numerous researchers have undertaken efforts to improve the manner by which high-order techniques may be constructed, with the aim of improving their accuracy while enhancing their stability and performance characteristics.

In this vein, the Discontinuous Galerkin (DG) method was developed because of its favorable attributes; these have led to its acceptance as one of the most widely relied upon high-order methods for solving the Navier–Stokes equations. The method itself was introduced in the context of the neutron transport problem by Reed and Hill [3], analyzed by LaSaint and Raviart [4] and then extended and popularized in the fluid dynamics community by Cockburn, Shu, Bassi, Rebay, and others (see [5–9], and the references therein).

One of the essential characteristics of the DG approach lies in its dependence on the Galerkin method to approximate a partial differential equation (PDE) that applies to a finite element. The corresponding PDE is subsequently converted into a series of ordinary differential equations (ODEs) that can be solved by standard methods.

Alternative approaches that seek to achieve high-order accuracy rely on differential forms. These may be exemplified by the pioneering work on the staggered-grid spectral method [10], as well as the spectral difference [11,12] and spectral volume approaches [13], which have been complemented by the elegant method of flux reconstruction [14,15,1] (FR), later evolving into the correction procedure via reconstruction [16–18] (CPR).

Among these high-order methods, different ways exist to appoint the degrees of freedom (DOFs) to each element at the cell-averaged or point-wise values, as well as their derivatives, which are later refreshed during the evaluation process. Although the Galerkin method and local reconstruction have been shown to provide formal avenues to derive the relevant ODEs in the context of the DG and differential approaches, the application of ADF to formulate the local ODEs will be used in this work as an alternative approach with particular benefits [1,2]. We further explore a generic ADF approach that contains arbitrary constants that can be specified in such a way to enhance the performance of the method to be reproduced. The characteristic attributes of this approach, such as simplicity, will constitute one of the main subjects of this article. In fact, one of the advantages of ADF implementation will be shown to be associated with its versatility in handling different DOF specifications.

It should also be noted that, in recent years, a well-developed constrained interpolation profile (CIP) with multi-moment finite volume (MFV) method has been developed (see Xie et al.[19] and the references therein). Apart from the cell-averaged value of a given element, MFV introduces additional DOFs on the element's edge and nodal points. The ability to share this supplementary information with neighboring elements transforms MFV into a more efficient scheme for saving the number of DOFs compared to other high-order methods of comparable accuracy. Pursuant to this approach, the sharing of additional DOFs within the context of continuity leads to the enhancement of the scheme's robustness. In fact, a similar concept may be attributed to the Active Flux (AF) method [20,21], where the unknown values at edge-based flux points are treated as independent DOFs and updated at every time step.

Because nodal points undergo the highest sharing rate, being shared by more elements than edges, it proves more efficient to increase the amount of information that is being communicated with a given element by placing all additional DOFs on the nodes only. As such, it is possible to augment the nodal information and extend the MFV and AF approaches by adding not only the unknown functional values at the nodal points, but also their derivatives. In this process, the updating of cell-averaged values may be accomplished in a manner that mirrors the traditional finite volume (FV) approach, thus guaranteeing the conservation of the scheme.

In practice, the manner by which additional information is updated on nodal points and edges constitutes the most distinguishing features in the MFV and AF schemes. We presently rely on an ADF procedure and set the integral domain to encompass all of the elements surrounding the point in question. This increases the radius of influence, as it were, that accompanies each update. Our nodal updating procedure may hence be likened to the case of an overlapped DG, where nodal values and derivatives can provide sufficient information for the high-order reconstruction of the unknown quantity in each element. As for the order of the "DG on the node," it is no longer constrained by the DOFs on the nodal point itself. The nodal updating becomes comparable to the PnPm procedure [22]. Furthermore, since the precision of the method may be improved by increasing the amount of information that is assigned to the nodal points, we call this strategy a point-value enhanced finite volume method (PFV). As to the temporal updating, a conventional third-order total variation diminishing (TVD) Runge–Kutta scheme can be conveniently employed.

In this article, the approximate delta function is revisited and extended in Section 2 to comprise arbitrary constants. This is followed by applying the extended ADF to recover and generalize Taylor-based DG, nodal-based DG, and FR/CPR methods in Sections 3, 4, and 5, respectively. In Section 6, the ADF-based point value enhanced finite volume method is defined, implemented, and verified numerically. We retire in Section 7 with some conclusions and recommendations for future work.

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