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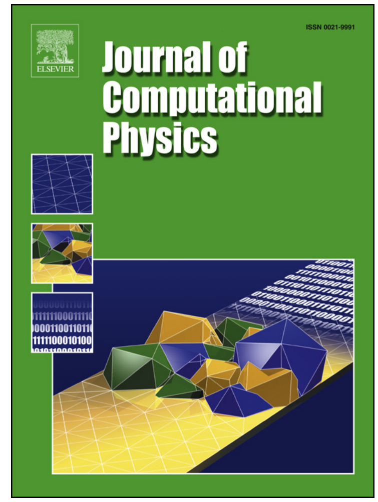

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On a free-surface problem with moving contact line: from variational principles to stable numerical approximations

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Abstract

We analyze a free-surface problem described by time-dependent Navier-Stokes equations. Surface tension, capillary effects and wall friction are taken into account in the evolution of the system, influencing the motion of the contact line – where the free surface hits the wall – and of the dynamics of the contact angle. The differential equations governing the phenomenon are first derived from the variational principle of minimum reduced dissipation, and then discretized by means of the ALE approach. The numerical properties of the resulting scheme are investigated, drawing a parallel with the physical properties holding at the continuous level. Some instability issues are addressed in detail, in the case of an explicit treatment of the geometry, and novel additional terms are introduced in the discrete formulation in order to damp the instabilities. Numerical tests assess the suitability of the approach, the influence of the parameters, and the effectiveness of the new stabilizing terms.

Keywords: capillary, Geometric Conservation Law, Arbitrary Lagrangian-Eulerian, generalized Navier boundary condition, contact angle

1. Introduction

The simulation of free-boundary problems is of major relevance in many fluid-dynamics applications, both at the large scale, like in the study of water waves [1] and the design of watercraft [2], and at the microscopic scale, e.g. in the microfluidics of capillary tubes [3, 4] or labs-on-a-chip [5, 6]. In these settings, the fluid under inspection interacts with other fluids or solids, and thus it is fundamental to correctly track the evolution of the interfaces between the different phases. Different approaches can be found, in the literature, for the modeling and the simulation of multiphase problems, and they can be classified in three main categories, depending on their treatment of the interfaces: the diffuse-interface models, the interface-capturing methods and the interface-tracking techniques. The phase-field model [7, 8, 5] is representative of the first category: regions occupied by different phases are identified by different integer values of a scalar function, and the interface has a finite thickness, spanning the region where this function smoothly passes from a level to another. This kind of smoothing of the interface allows an accurate physical characterization (including phase transitions) and helps in the development and the proof of theoretical results, but does not provide a sharp position of the interface. On the other hand, in interface-capturing methods, like the level-set method [9, 10] or the volume-of-fluid method [11, 12], a precise description of the interface is given at any time, as a codimension-1 manifold immersed in the domain. However, these methods require the solution of both the fluid phases separated by the interface, and at the discrete level it is crucial to properly handle the elements of the computational domain through which the interface passes, since the grid is not conforming to the interface. Eventually, the third category of methods includes the techniques to track the

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