



Compatible, energy conserving, bounds preserving remap of hydrodynamic fields for an extended ALE scheme

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ABSTRACT

From the very origins of numerical hydrodynamics in the Lagrangian work of von Neumann and Richtmyer [83], the issue of total energy conservation as well as entropy production has been problematic. Because of well known problems with mesh deformation, Lagrangian schemes have evolved into Arbitrary Lagrangian–Eulerian (ALE) methods [39] that combine the best properties of Lagrangian and Eulerian methods. Energy issues have persisted for this class of methods. We believe that fundamental issues of energy conservation and entropy production in ALE require further examination.

The context of the paper is an ALE scheme that is *extended* in the sense that it permits cyclic or periodic remap of data between grids of the same or differing connectivity. The principal design goals for a remap method then consist of total energy conservation, bounded internal energy, and compatibility of kinetic energy and momentum. We also have secondary objectives of limiting velocity and stress in a non-directional manner, keeping primitive variables monotone, and providing a higher than second order reconstruction of remapped variables.

In particular, the new contributions fall into three categories associated with: energy conservation and entropy production, reconstruction and bounds preservation of scalar and tensor fields, and conservative remap of nonlinear fields. The paper presents a derivation of the methods, details of implementation, and numerical results for a number of test problems. The methods requires volume integration of polynomial functions in polytopal cells with planar facets, and the requisite expressions are derived for arbitrary order.

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1. Introduction

From the very origins of numerical hydrodynamics in the Lagrangian work of von Neumann and Richtmyer [83], the issue of total energy conservation as well as entropy production has been problematic. Only in recent years have methods for conserving energy within the traditional spatially staggered Lagrange framework (SGH) become widespread. The lack of conservation was a consequence of staggered differencing in which internal energy naturally resided in the cell and kinetic energy at the nodes. The earliest work in 1D was due to Trulio and Trigger [77]. Conservation in the multi-dimensional SGH framework was eventually enabled by the development of so-called compatible energy hydro schemes [11,10,13,12,23]. In the last decade, cell-centered Lagrangian schemes (CCH) based on Godunov methods have appeared that naturally conserve total energy [29,57,16].

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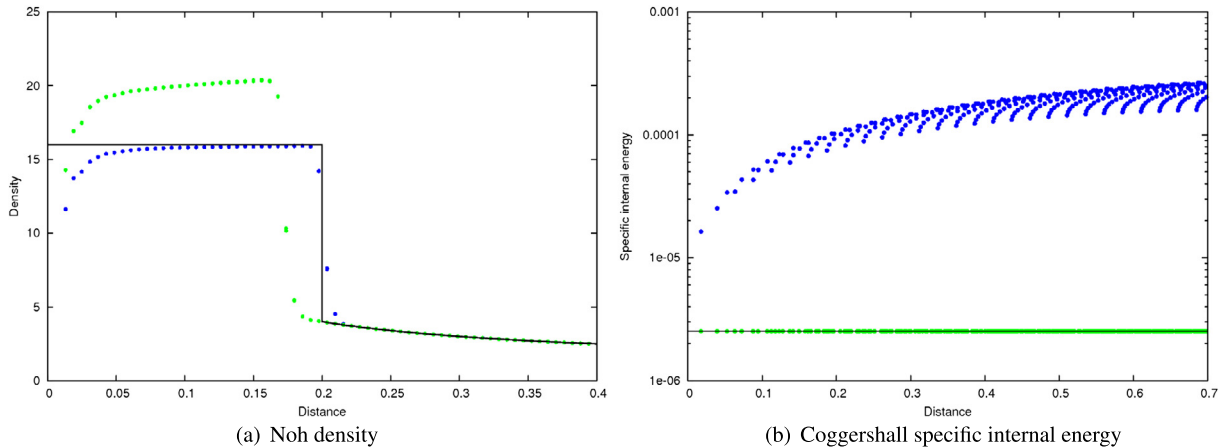


Fig. 1. Scatter plots for two ALE test problems that are discussed in detail later. Both simulations used a polar mesh. Figure (a) shows density vs. distance for the Noh problem (Section 6.1.2) at $t = 0.6$. The figure compares results from an internal energy (IE) conserving method (green) and a total energy (TE) conserving method (blue) with the analytic solution (black). Figure (b) shows specific internal energy on a logarithmic scale vs. distance for the Coggershall problem (Section 6.2.2) at $t = 0.5$. Again, the same IE and TE methods are compared with the analytic results. However, in (a), the TE method yields the best result, while in (b), the IE result is best. These calculations used a reconstruction method (designated CM) discussed later. (For interpretation of the colors in this figure, the reader is referred to the web version of this article.)

Because of well known problems with mesh deformation, Lagrangian schemes have since evolved into Arbitrary Lagrangian–Eulerian (ALE) methods [39] that combine the best properties of Lagrangian and Eulerian methods. Traditional ALE schemes *advect* material only between adjacent cells within a mesh that is incrementally adapted. The context of the paper is an *extended ALE* scheme that permits cyclic or periodic transfer of data between grids of the *same or differing connectivity*. Viewed in detail, the ALE procedure involves: (a) a Lagrange step, (b) mesh optimization, (c) determination of intersection volumes, (d) reconstruction of fields within donor cells, (e) a conservative remap, and finally (f) a repartition of kinetic and internal energy in acceptor cells. Although the overall context is more general, the focus of this paper is on the physics and numerics of the reconstruction (d) through repartition steps (f). In particular, the new contributions fall into three categories associated with: energy conservation and entropy production, reconstruction and bounds preservation of scalar and tensor fields, and conservative remap of nonlinear fields.

1.1. Energy and entropy remap

We believe that fundamental issues of energy conservation and entropy production have persisted for ALE methods and require further examination. Although total energy conservation is numerically straightforward, the literature addressing entropy production in ALE is sparse. We examine the fundamental entropy issues in Section 5. Although we discuss entropy production in a conceptual sense, entropy itself is not usually calculated. Rather, we use the term to imply the dissipation of kinetic energy into internal energy.

Internal energy (IE) method Early ALE, as well as Eulerian codes, were based on SGH and typically conserved momentum and internal energy, but not total energy, just as the Lagrangian codes did. In CCH as well as SGH, this can cause entropy errors that affect shock states and propagation velocities as demonstrated in test problems such as the Noh problem [63,18].

Total energy (TE) method An alternative strategy is to conservatively remap the total energy and calculate the internal energy by subtracting kinetic energy from the total energy. Total energy conserving methods can produce significant errors in problems involving isentropic flows such as the Coggershall problem [27].

The calculations presented in Fig. 1 as well as all figures throughout the paper were carried out with the same underlying code [14] with only algorithmic components varied. The Noh and Coggershall problems will be discussed in more detail in Sections 6.1.2 and 6.2.2. Here, we present a preview to highlight the energy issue. We use a center of mass reconstruction scheme described in the next section. Fig. 1(a) compares results from the Noh problem using an IE method with a TE method. The IE method is not conservative, produces a slow propagation velocity and overshoots the shock state, while the TE method matches the arrival and plateau quite well. Both methods show the expected wall heating near the origin [63]. Fig. 1(b) is a scatter plot of specific internal energy vs. distance for the Coggershall problem and again compares the IE and TE methods. In contrast to the Noh problem, the IE method gives the correct solution (uniform across the mesh), while the TE method is in error by nearly two orders of magnitude because of excess heating during remap.

The point, of course, is that neither IE nor TE methods solve both shock and smooth flow problems. In Section 5, we derive a new *compatible energy (CE)* method that resolves the energy issues.

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