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A multi-scale residual-based anti-hourglass control for compatible staggered Lagrangian hydrodynamics

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ABSTRACT

Hourglassing is a well-known pathological numerical artifact affecting the robustness and accuracy of Lagrangian methods. There exist a large number of hourglass control/suppression strategies. In the community of the staggered compatible Lagrangian methods, the approach of sub-zonal pressure forces is among the most widely used. However, this approach is known to add numerical strength to the solution, which can cause potential problems in certain types of simulations, for instance in simulations of various instabilities. To avoid this complication, we have adapted the multi-scale residual-based stabilization typically used in the finite element approach for staggered compatible framework. In this paper, we describe two discretizations of the new approach and demonstrate their properties and compare with the method of sub-zonal pressure forces on selected numerical problems.

1. Introduction

The Lagrangian methods are widely used for hydrodynamic simulations in many fields of physics due to their ability to naturally follow large deformations of the computational domain. Unfortunately, the moving computational mesh can degenerate during the Lagrangian motion. The solution then becomes distorted, or the simulation fails completely due to potentially non-convex or inverted computational cells. In general, this problem can have many different reasons, such as the presence of strong shears or vortexes in the solution. In this case, the standard purely Lagrangian approach is not suitable and one must switch to a more advanced technique, such as the Arbitrary Lagrangian–Eulerian (ALE) methods introduced in [1] or the Lagrangian methods employing curvilinear computational meshes [2–4].

On the other hand, the mesh degeneracies can arise differently than due to the solution features resulting from the physics, but due to the pathological properties of the numerical schemes, resulting typically from a nonphysically-high degree of freedom of the quadrilateral cells in the computational mesh. Such pattern in the computational mesh is usually termed as hourglass due to the typical shape of the computational cells under such motion. This pathological behavior of the Lagrangian schemes has been first observed in [5] and described many times afterward in different contexts, see for example [6–9].

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To control hourglass, many techniques have been developed, for an overview see the seminal paper [10]. The majority of such methods is based on an advanced formulation of the artificial viscosity, which is added to the solution and acts against the hourglass mode in the Lagrangian mesh motion. Another approach is based on enriching the solution by strain instead of viscosity, such as in the classical method [6]. Similarly, strain is added in the approach of sub-zonal pressure forces [11], which is the standard technique nowadays in the field of staggered compatible mimetic methods [12].

In this approach, the continuous function differential operators are replaced by the discrete function operators, mimicking the properties of the continuous ones [13]. The computational mesh motion is driven by the pressure gradient, which is discretized in this approach in the form of pressure forces acting from each cell to the adjacent nodes [12]. As the same forces act in the discretized energy equation, the entire numerical scheme is energy conserving no matter what the forces are. In principle, when the simulation needs to be enhanced by another physical phenomenon (such as viscosity, gravity, or elasticity), additional forces need to be constructed and added to the numerical scheme. In [11], it is shown how to split the computational cell into sub-zones and use the variance in the fluid density (and consequently pressure) in the cell to suppress the hourglassing mode in the form of sub-zonal pressure forces.

In the multi-material case, there are several material polygons present in a single computational cell [14,15]. In this case, the approach of sub-zonal pressure forces has to be generalized for multiple materials. The most straightforward way is the computation of the sub-zonal pressures by materials. This however leads to perform material reconstruction on a sub-zonal level, which significantly increases the computational cost and complexity of the Lagrangian solver algorithm – for example, one needs to propagate the material volume fractions over the Lagrangian step on the sub-zonal level either. Therefore, various simplifying assumptions are typically used which allow to use average cell value for the force construction, for example the uniform distribution of materials in the cell used in [16]. One of the examples is the mechanism described in [17], reformulating the sub-zonal pressure variation in terms of pressure derivative, which is proportional to the cell-average speed of sound, resulting from the particular multi-material closure model.

As the approach of sub-zonal pressure forces is based on supplemental numerical strength added to the solution, it can affect the solution in certain sensitive cases, such as instability growth studies. In such simulations, the formation of the instability is typically driven by a small vorticity in the fluid, generated by a pressure variance in case of Rayleigh–Taylor instability, velocity variance in case of Richtmyer–Meshkov instability, or non-uniform shear flow in case of Kelvin–Helmholtz instability. This small vorticity can be suppressed significantly by the additional strength, but resulting in decreasing the instability growth rate, which is crucial in certain types of applications (Inertial Confinement Fusion, for example).

On the other hand, in the context of the finite element methods, there exists the residual-based hourglass control mechanism [18–21], based on the variational multiscale (VMS) analysis of the discrete solution. In this approach, the residuals represent the deviation of the numerical solution from the analytic functions on a sub-cell level. The authors have shown that this fine-scale variance acts as a diffusive operator suppressing the hourglass modes in the solution. As it is based on adding numerical diffusion instead of numerical strength, it can be expected that its behavior in the mentioned tests will be better.

In this paper, we present a new hourglass-control mechanism for the staggered multi-material Lagrangian solver, based on the discretization of the pressure residual term from [20] in the staggered Lagrangian scheme. We have developed the discrete differential operators for two different ways of splitting the cell to sub-cells – the triangles corresponding to the cell edges or quadrilaterals corresponding to cell corners. Although both discretizations are presented for quadrilateral cells, the approach is independent of the particular mesh and can be simply generalized for general polygonal cells. Only the velocity divergence operator (joint for all cell materials in the standard staggered approach) is discretized on the sub-cell level, the rest of the pressure-residual term is constructed on the cell basis. This construction allows a simple generalization for multi-material cells.

The rest of this paper is organized as follows. In Section 2, the fine-scale residual formulation of the solution of the Euler equations in the Lagrangian coordinate system [20] is overviewed, resulting in the pressure residual term. The construction of discrete differential operators is presented in Section 3. After a brief overview of the standard staggered hydrodynamic scheme and the technique of sub-zonal pressure forces in Section 3.1, the construction of the pressure-residual forces in the triangular discretization is shown in Section 3.2 and in the sub-quad discretization in Section 3.3. Generalization of the pressure-residual forces for multi-material cells is described in Section 3.4. In Section 4, the new approaches are tested on a suite of standard numerical hydrodynamic tests. The whole paper is concluded in Section 5.

2. Derivation of pressure residual hourglass control

Our approach takes inspiration from the variational multiscale (VMS) hourglass control method originally proposed in [19,20], and discusses their relations to previous work in [22,23] and [24,25].

In Lagrangian coordinates, the system of Euler equations (in the updated-Lagrangian formulation) can be written as

$$\dot{\rho} + \rho \, \nabla \cdot \vec{u} = 0 \,, \tag{1}$$

$$\rho \, \dot{\vec{u}} + \nabla p = \vec{0} \,, \tag{2}$$

$$\rho \, \dot{\varepsilon} + p \, \nabla \cdot \vec{u} = 0 \,. \tag{3}$$