



A class of renormalised meshless Laplacians for boundary value problems



Josip Basic^a, Nastia Degiuli^{b,*}, Dario Ban^a

^a Faculty of Electrical Engineering, Mechanical Engineering and Naval Architecture, University of Split, Rudera Boskovicica 32, 21000 Split, Croatia

^b Faculty of Mechanical Engineering and Naval Architecture, University of Zagreb, Ivana Lucica 5, 10000 Zagreb, Croatia

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ABSTRACT

A meshless approach to approximating spatial derivatives on scattered point arrangements is presented in this paper. Three various derivations of approximate discrete Laplace operator formulations are produced using the Taylor series expansion and renormalised least-squares correction of the first spatial derivatives. Numerical analyses are performed for the introduced Laplacian formulations, and their convergence rate and computational efficiency are examined. The tests are conducted on regular and highly irregular scattered point arrangements. The results are compared to those obtained by the smoothed particle hydrodynamics method and the finite differences method on a regular grid. Finally, the strong form of various Poisson and diffusion equations with Dirichlet or Robin boundary conditions are solved in two and three dimensions by making use of the introduced operators in order to examine their stability and accuracy for boundary value problems. The introduced Laplacian operators perform well for highly irregular point distribution and offer adequate accuracy for mesh and mesh-free numerical methods that require frequent movement of the grid or point cloud.

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1. Introduction

Mesh-based methods for solving partial differential equations (PDEs) have achieved remarkable success and they have been successfully applied in various fields of engineering. Creating topologically correct and geometrically conforming mesh, which needs to be appropriate for an investigated problem, is time-consuming work and often requires user interventions within mesh generation steps. In addition, a simulation that requires large mesh deformations is difficult to maintain in geometrically complex problems, e.g. fluid-structure interaction. Deformations require cautious node movements or the re-meshing of deforming areas to avoid mesh tangling and loss of mesh regularity. An effective alternative to dealing with dynamic conforming meshes are mesh methods that capture the geometry of the free boundary. Examples include methods based on the adaptive grid refinement of Quadtree and Octree grids, for which re-meshing is straightforward and efficient [1]. Such methods have satisfactory parallel scalability and can be extended to perform massively parallel computations [2]. Another alternative is compact-support meshless or mesh-free methods that do not require any connectivity information. Solving PDEs in a meshless Lagrangian form can be used to naturally simulate advection, large deformations and free-surface, removing the need for mesh generation and complex deformation management.

* Corresponding author.

E-mail addresses: jobasic@fesb.hr (J. Basic), nastia.degiuli@fesb.hr (N. Degiuli), darioban@fesb.hr (D. Ban).

Reliable and accurate approximations of a gradient and a Laplacian are needed when solving various PDEs described with strong formulations: Navier–Stokes equations, the diffusion equation, the Helmholtz equation, the Schrödinger equation, the wave equation, and, of course, other general Laplace and Poisson equations. The Laplace operator, or the Laplacian, is defined as the divergence of the function gradient in Euclidean space, which is equivalent to the sum of unmixed second partial derivatives. The accuracy of discrete spatial operators is not the same on irregular grids as it is for corresponding operators on structured grids, i.e. the accuracy diminishes with an increase of mesh irregularity. In the finite volume (FV) method, the Laplacian used to model viscous forces is obtained by applying the Green–Gauss theorem, often with an explicit artificial dissipation added at each time step. De Foy and Dawes [3] have shown that the repeated action of a first-order accurate FV operator leads to zero-order accuracy, with errors up to 50% on irregular meshes. In addition, the positivity of coefficients is not even satisfied on regular grids, which leads to a lack of robustness when the operator is used for smoothing or solving Poisson's equation. Juretic et al. [4] have shown that the accuracy of the diffusion term is dependent on the interpolation scheme for skewed cells' faces, and on the discretisation procedure for surface normal gradients. The Laplacian in non-continuous form cannot guarantee the “divergence of gradient” identity, and Owen et al. [5] have shown that the discrete Laplacian should be used to obtain a stable velocity correction scheme that depends on the pressure Poisson equation. Gibou et al. [6] introduced the discretisation of the Laplacian on adaptively refined Quadtree/Octree grids with captured geometry by employing ghost values, which are obtained using various extrapolation schemes. Gibou and Min [7] have employed the introduced Laplacian to obtain the symmetric positive definite linear system for the second-order-accurate monolithic fluid–solid interaction (FSI) solver.

Various meshless methods are currently under active development. One of the main motivations is the recent advances in data-parallel algorithms and corresponding parallel-execution hardware, and the ease of applying such algorithms to methods that are based on compact support spheres. While there are many variations of meshless and hybrid-mesh-meshless methods, the following text summarises pertinent methods that have introduced discrete Laplacians.

The most popular meshless method is the smoothed particle hydrodynamics (SPH) method which represents quantities as discrete particles and uses a kernel function to smooth their volumetric contributions. However, SPH methods have a number of problems that require special fixes, which sometimes come at great cost, but do not eliminate all low-order inconsistencies. Brookshaw [8] and Monaghan and Gingold [9] have introduced smoothing kernel gradient-based SPH discretisations of the Laplacian, which still today are one of the most widely used formulas, although they are inconsistent and should be corrected [10–12]. Chaussonnet et al. [13] have shown how sensitive the accuracy of typically used SPH Laplacian schemes can be to the particle disorder and smoothing radius. Fatehi and Manzari [12] have reviewed typically used schemes for the second derivative using theoretical analysis, and have introduced first-order consistent second derivatives based on the renormalisation tensors.

The original Laplacian used in the moving particle semi-implicit (MPS) method was proposed by Koshizuka et al. [14]. Isshiki [15] successfully reproduced it via the Gauss divergence theorem. Zhang et al. [16] have shown its inconsistencies and the numerical difficulties that emerge when solving Poisson's equation, and introduced the enhanced version of the operator [17]. Ng et al. [18] have evaluated the accuracy of various MPS Laplacian models by solving Poisson's equation subjected to both Dirichlet and Neumann boundary conditions, and proposed a more general model with the altered kernel function. They deduced that refining the grid spacing while retaining the number of neighbour interpolation points is not applicable due to the numerical errors, and, moreover, the optimal number of neighbours is dependent on the degree of grid irregularity. Ikari et al. [19] have proposed a corrected higher order Laplacian scheme, which is derived by taking the divergence of the corrected LS gradient model. The resulting correction tensor is used in combination with the first- and second-order derivative of a smoothing kernel function, which produces more accurate results. Ma et al. [11] have reviewed the meshless pressure projection procedure by solving Poisson's equation with the most popular discretisations of the Laplacian, often adopted in the incompressible smooth particle hydrodynamics (ISPH) method and moving particle semi-implicit (MPS) methods. Recently, Tamai et al. [10] have tested the accuracy and consistency of meshless discretisation schemes for the Laplacian, and employed the studied schemes to find the solution to Poisson's equation in strong formulation with the imposed Dirichlet boundary conditions.

Huang et al. [20] have proposed that the Laplacian for the kernel gradient free (KGF) SPH method is obtained by carrying out two first derivative operations, i.e. by means of two inversions of symmetric tensors of size $(d + 1) \times (d + 1)$, where d is the number of dimensions. The computational cost of the two-pass Laplacian done for each particle is inconvenient for dynamic meshless simulations, and it is difficult to implement boundary conditions. Lei and Peng [21] have proposed an approximate Laplacian model, which is sensitive to particle distribution without using a large smoothing radius, thus requiring frequent particle shifting for dynamic simulations.

In the finite point-set method (FPM) [22], a function and its derivatives are approximated by the second-order moving least squares (MLS) method for each point in a domain. Higher order derivatives are obtained directly since the Hessian coefficients are included while solving the MLS problem. The method proved to be accurate when solving Poisson's equation for incompressible flows [23,24], although the vector of ten unknowns solved for each point in three dimensions is not computationally efficient for non-static point distribution, e.g. for transient Lagrangian applications.

In the particle strength exchange (PSE) method, the Laplacian is approximated by an integral operator, which is then transformed into the discrete form by a quadrature over the particles [25]. The symmetry in the PSE method can conserve properties that are inherent when two particles “exchange strength” with one another. On the other hand, the PSE method

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