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Short note

Stability analysis of a partitioned iterative method for steady free surface flow

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1. Introduction

This note considers the steady free surface (FS) flow problem as encountered in the paper by van Brummelen et al. [1]. In that paper, steady flow of water in a two-dimensional slice of an infinitely wide open channel with a particular bottom wall is calculated as the first step in the development of a 3D surface fitting method for steady flow around ships. In these water–air flows, the influence of air is usually negligible due to the large difference in density. Contrary to surface capturing methods which are typically multiphase techniques (such as the volume-of-fluid method), fitting methods usually consider only the water phase. The latter approach requires appropriate FS boundary conditions. The dynamic boundary condition (DBC) used here assumes that the pressure is constant (atmospheric) at the FS and the shear stresses are zero. The kinematic boundary condition (KBC) states that the FS is impermeable.

Fitting methods typically consist of an iterative process with two steps that are repeated until convergence is reached in each time step: first calculation of the flow field with a fixed FS position and suitable boundary conditions, and then update of the FS position. Often the DBC is used in the first step, the KBC in the second [2,3]. Using the KBC for updating the surface results in a time-stepping method, which is not efficient for steady cases due to the large number of time steps before transient phenomena have disappeared. Van Brummelen [1] describes a steady iterative fitting method which uses the DBC for the surface update and a combined boundary condition (KBC + DBC) in the flow solver. This method is efficient, but because the pressure gradient and the vertical velocity are both calculated implicitly at the FS [4], it requires access to the source code of a coupled solver, making it less flexible. Combining a steady iterative approach with a black box solver requires that the boundary conditions in the flow solver are easy to implement (only KBC) and that the update is time-independent (DBC).

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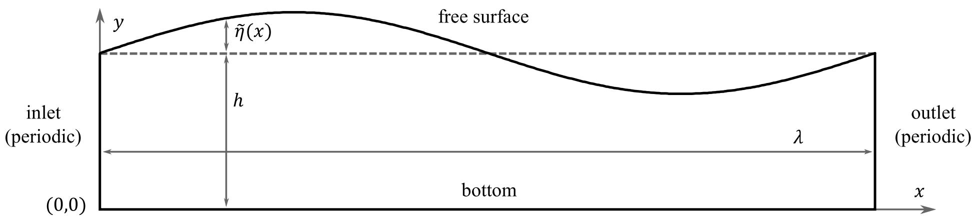

Fig. 1. Domain studied in modal analysis.

A modal analysis is carried out in this note to investigate the relation between position and pressure perturbation errors at the FS for the latter boundary conditions. A von Neumann stability analysis of Gauss–Seidel iterations is then performed, which iterate between solving the flow and updating the surface.

2. Modal analysis

The case studied in the modal analysis is inviscid FS flow over a flat bottom, which has a horizontal surface as solution. A small perturbation $\tilde{\eta}$ of the FS height is considered, which can be split into its Fourier-modes a_0, $a_i \sin ikx$ and $b_i \cos ikx$ with $i \in \mathbb{N}_0$ and $k = 2\pi/\lambda$ the wavenumber. Sine and cosine modes will lead to the same flow field shifted over $\lambda/4$, so only sines are studied. The constant mode a_0 is not of interest: it leads to a constant pressure at the FS, and is therefore also a solution of the FS flow problem. The problem studied is shown in Fig. 1: a periodic flow with FS position error

$$\tilde{\eta}(x) = a \sin kx \tag{1}$$

and average inflow-velocity U. The bottom and FS are considered impermeable free-slip walls.

The modal analysis is performed in two ways. A potential flow solution is derived analytically, relating the FS pressure error $\tilde{p}$ to the FS position error $\tilde{\eta}$. The results are then checked against numerical simulations using a finite volume flow solver.

2.1. Potential flow solution

The flow field is derived for a small perturbation $\tilde{\eta}$, resulting in two conditions on the error amplitude:

$$a \ll \lambda \qquad \text{and} \qquad a \ll h \tag{2}$$

The complex potential w of this flow is

$$w = \frac{aU}{\sinh kh}\cos(kz) + Uz = \phi + i\psi \qquad \text{with} \qquad z = x + iy \tag{3}$$

The velocity potential ϕ and stream function ψ are defined as

$$\boldsymbol{u} = [u, v] = \left[\frac{\partial\phi}{\partial x}, \frac{\partial\phi}{\partial y}\right] = \left[\frac{\partial\psi}{\partial y}, -\frac{\partial\psi}{\partial x}\right] \tag{4}$$

with $\boldsymbol{u}$ the velocity vector. It can be proven that w gives the correct flow field by verifying that the velocity is indeed periodic between inlet and outlet and that ψ is constant along the bottom and the free surface. The former is straightforward to prove, the latter is done in [5] for a similar flow field.

The velocity $\boldsymbol{u}$ is found from (4), the pressure p is derived from the Bernoulli equation which reduces to

$$p = -\frac{\rho\boldsymbol{u}^2}{2} - \rho g y + C = -\frac{\rho U^2}{2}\left[1 + \left(\frac{ak}{\sinh kh}\right)^2\left(\sinh^2 ky + \sin^2 kx\right) - 2\frac{ak}{\sinh kh}\cosh ky \sin kx\right] - \rho g y + C \tag{5}$$

The FS pressure error $\tilde{p}$ is found by substituting $y = h + \tilde{\eta}$ and choosing the constant C so that p is zero at the intersection between the FS and the inlet (coordinate $[0, h]$), giving

$$\tilde{p} = -\frac{\rho U^2}{2}\left[1 + \left(\frac{ak}{\sinh kh}\right)^2\left(\left(1 + \mathcal{O}\left(\frac{a}{\lambda}\right)\right)\sinh^2 kh + \sin^2 kx\right) - 2\frac{ak}{\sinh kh}\left(1 + \mathcal{O}\left(\frac{a}{\lambda}\right)\right)\cosh kh \sin kx\right] - \rho g y + C \tag{6}$$

$$\approx -\frac{\rho U^2}{2}\left[1 + (ak)^2 + \left(\frac{ak \sin kx}{\sinh kh}\right)^2 - 2\frac{ak \sin kx}{\tanh kh}\right] - \rho g\,(h + a \sin kx) + \frac{\rho U^2}{2}\left(1 + (ak)^2\right) + \rho g h \tag{7}$$