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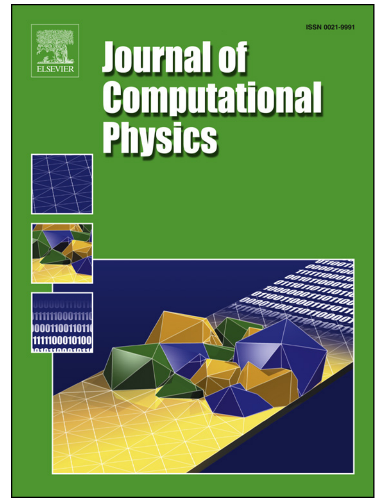

The Ericksen Model of Liquid Crystals with Colloidal and Electric Effects

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The Ericksen Model of Liquid Crystals with Colloidal and Electric Effects

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Abstract

We present a robust discretization of the Ericksen model of liquid crystals with variable degree of orientation coupled with colloidal effects and electric fields. The total energy consists of the Ericksen energy, a weak anchoring (or penalized Dirichlet) energy to model colloids, and an electrical energy for a given electric field. We describe our special discretization of the total energy along with a method to compute minimizers via a discrete quasi-gradient flow algorithm which has a strictly monotone energy decreasing property. Numerical experiments are given in two and three dimensions to illustrate that the method is able to capture non-trivial defect patterns, such as the Saturn ring defect. We conclude with a rigorous proof of the Γ-convergence of our discrete energy to the continuous energy.

Keywords: liquid crystals, finite element method, gamma-convergence, gradient flow, line defect, plane defect, Saturn ring defect
2000 MSC: 65N30, 49M25, 35J70

1. Introduction

This paper presents a method for solving the Ericksen model of liquid crystals [1, 2], with additional effects due to colloidal domains and electric fields. Liquid crystals are a work-horse technology enabling electronic displays [3, 4, 5], for instance. Moreover, they have a host of potential applications in material science [6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21]. One avenue is to use external fields (e.g. electric fields) and colloidal dispersions to build new materials through directed self-assembly [22, 7, 12, 23, 24, 25, 26, 27, 28, 29, 16, 19, 30, 31].

A significant amount of mathematical analysis has been done on liquid crystals [32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44]. Moreover, a host of numerical methods have been developed for statics and dynamics [45, 46, 47, 48, 49, 50, 51]. In particular, the methods

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