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The use of proper orthogonal decomposition (POD) meshless RBF-FD technique to simulate the shallow water equations

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ABSTRACT

The main aim of this paper is to develop a fast and efficient local meshless method for solving shallow water equations in one- and two-dimensional cases. The mentioned equation has been classified in category of advection equations. The solutions of advection equations have some shock, thus, especial numerical methods should be employed for example discontinuous Galerkin and finite volume methods. Here, based on the proper orthogonal decomposition approach we want to construct a fast meshless method. To this end, we consider shallow water models and obtain a suitable time-discrete scheme based on the predictor-corrector technique. Then by applying the proper orthogonal decomposition technique a new set of basis functions can be built for the solution space in which the size of new solution space is less than the original problem. Thus, by employing the new bases the CPU time will be reduced. Some examples have been studied to show the efficiency of the present numerical technique.

1. Introduction

A wide variety of physical and natural phenomena such as sound, heat, electrostatics, electrodynamics, fluid flow or elasticity have been described using partial differential equations (PDEs) [72,73]. We refer the interested reader to [104] for various applications of partial differential equations in sciences and engineering and also for some approaches in obtaining their solutions.

The shallow water (SW) equations can be considered as a simplification of the Navier–Stokes equations [102]. In the shallow water models, the horizontal wave length is much larger than the depth of the fluid [102]. For this reason, some special numerical methods such as discontinuous Galerkin, finite volume and adaptive moving mesh methods can be used for solving the advection problems.

1.1. A brief review on SWs equations

In the current paper, we consider three boundary value problems in water science:

1. **The 1D and 2D shallow water equations without friction term:**
   Trahan and Dawson [101] investigated a second-order, local time stepping procedure within a Runge–Kutta discontinuous Galerkin (RKDG) framework to solve the shallow water equations. The dam-break problems in one- and

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two-dimensional shallow water equations are attractive problems in numerical analysis and many investigations have been done on these problems. The public solution for the dam-break problem has been developed by Cozzolino and his co-workers [30].

Benkhaldoun and his co-workers [2] developed a new stabilized meshless method based on the radial basis functions for the numerical solution of convection-dominated flow problems. This class of problems includes viscous Burgers equations and incompressible Navier–Stokes equations at high Reynolds numbers. A new finite volume method has been proposed in [3] based on the predictor stage and a corrector stage for the numerical solution of shallow water equations for either flat or non-flat topography. The main aim of [4] is to propose slope limiters in meshless radial basis functions for solving nonlinear equations of conservation laws with flux function that depends on discontinuous coefficients as the method is based on the local collocation formulation. Authors of [1] presented the similarity solution to the Riemann problem of the one dimensional shallow water equations (SWE) with a bottom step discontinuity. Using the class of fractional-step procedures, a simple and accurate projection finite volume method is developed in [5] for solving shallow water equations in two-space dimensions. A new class of finite volume method is presented in [6] for solving shallow water flows in porous media on unstructured triangular grids in which the method consists of two stages which can be interpreted as a predictor-corrector procedure.

Navas-Montilla and Murillo [82] proposed an arbitrary accurate derivatives Riemann problem (ADER) type finite volume numerical scheme as an extension of a first-order solver with source terms. Li and his co-workers [71] proposed a numerical procedure for shallow water equations with a source term as the equations admit steady state solutions in which the non-zero flux gradient is exactly balanced by the source term. Xing [106] developed well-balanced discontinuous Galerkin approaches for the shallow water system. Sun and his co-authors [95] applied the meshless local RBF differential quadrature (LRBFDQ) technique to simulate the shallow water equations (SWE).

A well-balanced, spatially arbitrary with high order accurate discontinuous Galerkin scheme is proposed by Tavelli and Dumbser [100]. Canestrelli and his co-workers [19] studied on finite volume method for the 2D shallow water equations. Dumbser and Casulli [43] developed a spatially arbitrary high-order, semi-implicit spectral discontinuous Galerkin (DG) scheme for the shallow water equations.

The main aim of the current paper is to develop a combined RBF-FD approach to solve some shallow water equations. The RBF-FD approach is constructed by combining radial basis functions concept and finite difference method. In the finite difference technique, the corresponding weights can be obtained by using local polynomial approximations. Also, radial basis functions can be chosen instead as basis functions translates of radially symmetric functions. Thus, combination of radial basis functions with finite difference approach leads to radial basis function-generated FD formulas. Furthermore, all approximations again local, but nodes can now be placed freely.

One local meshless method is smoothed particle hydrodynamics (SPH) that is presented in [52,131]. The SPH technique is a computational method used for simulating the dynamics of continuum media, such as solid mechanics and fluid flows. The SPH method is a mesh-free Lagrangian method where the coordinates move with the fluid, and the resolution of the method can easily be adjusted with respect to variables such as the density. The SPH method is based on the dividing the fluid into a set of discrete elements that they are well-known as "particles". These particles have a spatial distance over which their properties are "smoothed" by a kernel function. This means that the physical quantity of any particle can be obtained by summing the relevant properties of all the particles which lie within the range of the kernel. Also, the SPH method is employed for the shallow water equation. The interested readers can find more information on SPH method in [118].

Wei and et al. [119] applied the SPH method to investigate the impact of a tsunami bore on simplified bridge piers in this study. This work was motivated by observations of bridge damage during several recent tsunami events. The main aim of [120,121] is apply the numerical model of GPUSPH, an implementation of the weakly compressible Smoothed Particle Hydrodynamics method on graphics processing units, to investigate tsunami forces on bridge superstructures and tsunami mitigation on bridges by using a service road bridge and an offshore breakwater. Authors of [122] investigated vorticity generation by short-crested wave breaking by using the mesh-free Smoothed Particle Hydrodynamics model.

Katta and his co-workers [68] developed a central-upwind finite volume (CUFV) scheme for solving shallow water model on a nonorthogonal equiangular cubed-sphere grid. High-order spatial discretization based on weighted essentially non-oscillatory (WENO) scheme is considered in [68].

Cotter and Thuburn [29] described discretisations of the shallow water equations on the sphere using the framework of finite element exterior calculus. They presented [29] two formulations as follows:

- "primal" formulation in which the finite element spaces are defined on a single mesh;
- "primal-dual" formulation in which finite element spaces on a dual mesh are used.

Felcman and Kadrnka [47] applied the moving mesh method to the shallow water equations. An upwind weighted essentially non-oscillatory scheme for the solution of the shallow water equations on generalized curvilinear coordinate systems is proposed by Gallerano and his co-workers [53]. Bistrian and Navon [13] proposed a framework for dynamic mode decomposition (DMD) of 2D flows, when numerical or experimental data snapshots are captured with large time steps. Jiang and Zhang in [65] developed new Krylov implicit integration factor-WENO methods to deal with both semilinear and fully nonlinear advection-diffusion-reaction equations.