# A high-accuracy Eulerian gyrokinetic solver for collisional plasmas 

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#### Abstract

We describe a new approach to solve the electromagnetic gyrokinetic equations which is optimized for accurate treatment of multispecies Fokker-Planck collisions including both pitch-angle and energy diffusion. The new algorithm is spectral/pseudospectral in four of the five phase space dimensions, and in the fieldline direction a novel 5 th-order conservative upwind scheme is used to permit high-accuracy electromagnetic simulation even in the limit of very high plasma $\beta$ and vanishingly small perpendicular wavenumber, $k_{\perp} \rightarrow 0$. To our knowledge, this is the first pseudospectral implementation of the collision operator in a gyrokinetic code. We show that the new solver agrees closely with GYRO in the limit of weak Lorentz collisions, but gives a significantly more realistic description of collisions at high collision frequency. The numerical methods are also designed to be efficient and scalable for multiscale simulations that treat ion-scale and electron-scale turbulence simultaneously.


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## 1. Introduction

For gyrokinetic studies of plasma microturbulence in the H -mode pedestal and L-mode edge, collisions are expected to play a more critical role than in the core due to the lower electron and ion temperatures, and correspondingly higher collision frequencies. Here the $\delta f$ approximation is formally valid for electron-scale fluctuations as well as for ion-scale fluctuations where the ion gyroradius is small compared to the equilibrium length scale, as long as the simulation region is far enough away from the separatrix so that orbit loss is not significant. The dominant collisional process in gyrokinetics is the detrapping of electrons, which reduces trapped-electron drive and thereby lowers linear instability growth rates. A very weak, secondary collisional process is the ion-ion collisional damping of zonal flows. We believe it is fair to suggest that existing Eulerian gyrokinetic codes use a numerical mesh that is motivated by, and optimized for, the collisionless equations rather than for accurate treatment of the collision operator. Thus, for pedestal-relevant studies, a significant improvement in gyrokinetic solvers may be realized by including more advanced gyrokinetic collision operators together with numerical algorithms optimized for their discretization in the strong-collisionality regime.

In neoclassical transport studies, comparative studies have shown that model operators can yield inaccuracies - compared with the full linearized Fokker-Planck operator - as large as $10-15 \%$ for the neoclassical particle fluxes and 20-30\% for the neoclassical ion energy fluxes [1]. On the other hand, gyrokinetic studies of the accuracy and limitations of commonly used model operators have been much more limited in scope. Early nonlinear gyrokinetic simulations included collisions

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Fig. 1. GYRO irregular $(\theta, \xi)$ mesh (a) showing constant-of-motion alignment, versus CGYRO regular ( $\theta, \xi$ ) mesh (b). In the GYRO mesh, dashed curves denote contours of constant $\mu$, such that a given collisionless unperturbed particle orbit is confined to single curve. The GYRO mesh is ideal for the collisionless case, whereas the CGYRO mesh becomes progressively more suitable for discretization of collisional plasmas. The shaded area corresponds to the trapped-particle region $\xi^{2}<1-B(\theta) / B(\pi)$.
with a relatively simple Lorentz pitch-angle scattering operator. Advanced operators including energy diffusion have recently been implemented in GS2 [2], GENE [3], and GKW [4]. However, the accuracy of the model operators and the convergence properties of their numerical implementations have not been well-explored. With GS2, it was shown in simplified Z-pinch geometry that energy diffusion suppresses the short wavelength structure beyond that due to pitch angle scattering alone [2]. Some detailed studies in tokamak geometry were done with GKW, showing that energy diffusion enhances the stabilization of trapped electron modes (TEM) [4].

Motivated by these considerations, a new gyrokinetic solver CGYRO has been developed for more accurate treatment of advanced collision operators in high-collisionality regimes. Building on experience with the neoclassical code NEO [5,1], CGYRO uses the $(\xi, v)$ coordinate system, where $\xi=v_{\|} / v$ is the cosine of the pitch angle, and $v$ is the speed. These coordinates simplify the analytic form of the collision operator and thereby permit the implementation of high-order numerical schemes to treat multi-species collisions and energy diffusion. As early as 1957 [6], these coordinates were pointed out as being particularly useful for respresentation of the Fokker-Planck collision operator.

As already noted, existing gyrokinetic codes use velocity-space coordinates optimized for the collisionless problem. For example, GYRO [7] and GS2 [8,9] use $(\lambda, \varepsilon)$, where $\lambda=v_{\perp}^{2} /\left(v^{2} B\right)$ and $\varepsilon=m_{a} v^{2} /\left(2 T_{a}\right)$. These coordinates are in fact unperturbed, collisionless constants of motion. The main advantage of the constant-of-motion coordinates is that the poloidal $(\theta)$ discretization is aligned with particle orbits, so that there is no need to take $\theta$-derivatives across the trapped-passing (TP) boundary. This feature of the ( $\lambda, \varepsilon$ ) coordinate system is illustrated in Fig. 1a, and compared with the ( $\xi, v$ ) system in Fig. 1b. For early - typically collisionless - gyrokinetic simulation, this was viewed as a critical feature of the algorithm since the collisionless electron distribution function is not continuous across the TP boundary. Conversely, the significant disadvantage of these coordinates is that it is very difficult to accurately discretize the collision operator since the constant-of-motion coordinates map to an irregular grid in $(\xi, \theta)$. To this end, GYRO uses a radial basis function interpolant in the $(\xi, \theta)$ plane [10]. Relatedly, the codes GENE [11] and GKW [12] use ( $\mu, v_{\|}$) coordinates, where $\mu=v_{\perp}^{2} /(2 B)$. The latter coordinates are the standard ones used for the Lie-transform derivation of the collisionless gyrokinetic equations [13]. Once again, although convenient for the collisionless case, $\left(\mu, v_{\|}\right)$coordinates are not ideal for evaluating the collision operator because they generate cross-derivatives, and lead to issues with constructing suitable boundary conditions. If instead we restrict the domain of interest to collisional plasmas for which the electron distribution is expected to be sufficiently smooth across the TP boundary, then the $(\xi, v)$ coordinates make it possible to implement elegant and spectrally-accurate discretization schemes. In this way, CGYRO builds upon the success of NEO by implementing spectrally-accurate collision operators in $(\xi, v)$ coordinates to describe inter-species collisions between particles with arbitrary masses.

The rest of this paper is organized as follows. In Section 2, the basic simulation equations, including the collision models, are described. In Section 3, details of the numerical discretization algorithms are given. Numerical results are presented in section 4, including benchmarks with GYRO and a comparison of reduced collision models on gyrokinetic nonlinear transport.

## 2. Theoretical formulation and simulation equations

### 2.1. Equilibrium geometry

We adopt the non-orthogonal field-aligned coordinate system ( $\psi, \theta, \alpha$ ), together with the Clebsch representation for the magnetic field [14]:

$$
\begin{equation*}
\mathbf{B}=\nabla \alpha \times \nabla \psi \quad \text { such that } \quad \mathbf{B} \cdot \nabla \alpha=\mathbf{B} \cdot \nabla \psi=0 . \tag{1}
\end{equation*}
$$

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