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Accurate computation of surface stresses and forces with immersed boundary methods

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ABSTRACT

Many immersed boundary methods solve for surface stresses that impose the velocity boundary conditions on an immersed body. These surface stresses may contain spurious oscillations that make them ill-suited for representing the physical surface stresses on the body. Moreover, these inaccurate stresses often lead to unphysical oscillations in the history of integrated surface forces such as the coefficient of lift. While the errors in the surface stresses and forces do not necessarily affect the convergence of the velocity field, it is desirable, especially in fluid–structure interaction problems, to obtain smooth and convergent stress distributions on the surface. To this end, we show that the equation for the surface stresses is an integral equation of the first kind whose ill-posedness is the source of spurious oscillations in the stresses. We also demonstrate that for sufficiently smooth delta functions, the oscillations may be filtered out to obtain physically accurate surface stresses. The filtering is applied as a post-processing procedure, so that the convergence of the velocity field is unaffected. We demonstrate the efficacy of the method by computing stresses and forces that converge to the physical stresses and forces for several test problems.

1. Introduction

Immersed boundary (IB) methods are attractive for simulating flows around moving or deforming bodies, due in large part to their ability to treat the immersed body and the flow domain with separate grids. The use of different grids removes the need for remeshing, which is often computationally expensive. The original IB method of Peskin introduced a singular source term in the momentum equations that imposed the stresses from the immersed body onto the flow grid [1]. In that work, the surface stresses were derived using a specific constitutive law.

A different set of IB methods retains the use of a singular source term to impose the surface stresses, but derives these stresses using velocity boundary conditions rather than by directly linking them to deformation of the solid [2–10]. Because they are derived from the boundary conditions on the immersed body, we refer here to these IB methods as surface velocity-based IB methods. These methods produce surface stresses that are poor representations of the physical surface stresses. A subset of these also produce unphysical oscillations in time traces of surface force quantities such as the coefficients of lift and drag, since they enforce the boundary constraint approximately rather than explicitly [2–4]. Yang et al. [5] reduced the unphysical oscillations in these surface force quantities, but to our knowledge the inaccuracies in the surface stresses have not been addressed. This is likely due to the fact that the velocity field converges in spite of these

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erroneous surface stresses, so surface velocity-based IB methods may be used without modification for problems where accurate knowledge of the surface stresses is not required.

However, correct information about surface stresses and forces is important in many applications, such as characterizing the performance of wings and aerodynamic bodies in unsteady flows, understanding and controlling flow separation around bluff bodies, and simulating fully coupled flow–structure-interaction (FSI) problems with deforming bodies. In this work, we characterize and remedy the spurious surface stresses and forces obtained by surface-velocity based IB methods.

It should be noted that there is a class of IB methods called "sharp-interface" methods, which includes ghost-cell [11], cut-cell [12], ghost-fluid [13], and immersed interface methods [14]. While spurious surface stress and force oscillations have been observed for a subset of these methods [15,16], their cause and remedy are different from what is presented in the current work [16]. A key distinction between this subset of sharp-interface methods and the methods considered here is the use of local flow reconstructions that obviate the need for a singular source term in the momentum equations.

We restrict our attention to methods that contain a singular source term in the momentum equations, and that compute surface stresses and forces using that term. We show that, for any choice of smeared delta function, the equation for the surface stresses is an integral equation of the first kind whose ill-posedness leads to an inaccurate representation of the high frequency components of the surface stresses. The error in these high frequency components was also observed by Kallemov et al. [10] for a six-point delta function. We demonstrate that there is an inverse relation between the smoothness of the smeared delta function and the amplitude of the high frequency components for the physically correct stress. Thus, when sufficiently smooth delta functions are selected, the high-frequency components that are erroneously amplified when solving the integral equation may be effectively filtered out of the solution without damaging the overall surface stress. By contrast, filtering out the incorrect high frequency components for insufficiently smooth smeared delta functions obscures important physical information.

We develop an efficient filtering technique for penalizing the erroneous high frequency stress components. The filtering procedure is performed as a post-processing step, so the convergence of the velocity field is unaffected. We demonstrate that, for all smeared delta functions considered, the filtered stresses are better approximations to the physical stresses than their unfiltered counterparts. However, because of the aforementioned inverse relationship between the smoothness of the smeared delta function and the magnitude of the high frequency components required to represent the physical stresses, this filtering procedure only provides convergent surface stresses when applied to sufficiently smooth smeared delta functions. These results are illustrated for several problems using the immersed boundary projection method (IBPM) of Colonius and Taira [8].

2. Demonstrating and resolving inaccurate computation of source terms for a model problem

The difficulty in solving integral equations of the first kind that arise from surface velocity-based IB methods is illustrated and remedied for a model problem in this section. Section 3 will demonstrate that the same type of integral equation arises from the Navier–Stokes equations. Thus, the same techniques developed here may be used to compute surface stresses and forces that arise in fluid flows.

The model problem considered is the Poisson equation for an unknown function ψ on a 2D square domain $\Omega = \{\mathbf{x} = [x, y]^T : |x|, |y| \leq 1\}$ with an unknown singular source term f that takes nonzero values on an immersed surface denoted by Γ:

$$\begin{aligned} \nabla^2\psi(\mathbf{x}) &= -\int_\Gamma f(\boldsymbol{\xi}(s))\delta(\mathbf{x}-\boldsymbol{\xi}(s))ds \\ \psi(\mathbf{x}) &= \psi^{\partial\Omega}(\mathbf{x}), \ \mathbf{x}\in\partial\Omega \\ \int_\Omega \psi(\mathbf{x})\delta(\mathbf{x}-\boldsymbol{\xi}(s))d\mathbf{x} &= \psi^\Gamma(\boldsymbol{\xi}(s)) \end{aligned} \tag{1}$$

where s is a variable that parametrizes the IB (*e.g.*, arc length), $\boldsymbol{\xi}(s)$ is the Lagrangian coordinate of a given point on the IB, $\partial\Omega$ is the boundary of the domain Ω, $\psi^{\partial\Omega}(\mathbf{x})$ is a function of prescribed values for ψ on $\partial\Omega$, and $\psi^\Gamma(\boldsymbol{\xi}(s))$ is a function defined on the immersed body. Note that the delta function $\delta(\mathbf{x}-\boldsymbol{\xi}(s))$ is used to relate quantities between the immersed surface and the solution domain. An error analysis of numerical solutions to (1) has been performed in the case where f is prescribed [17,18]. To mirror surface velocity-based IB methods, we leave f as an unknown that is solved by explicitly incorporating the third equation as a boundary constraint.

We take Γ to be a circle of radius 1/2 centered at $\mathbf{x}=0$, $\psi^{\partial\Omega}(\mathbf{x}) = 1-\frac{1}{2}\log(2|\mathbf{x}|)$, and $\psi^\Gamma(\boldsymbol{\xi})=1$. The exact solution to (1) is then

$$\psi_{ex}(\mathbf{x}) = \begin{cases} 1, & |\mathbf{x}| \leq \frac{1}{2} \\ 1-\frac{1}{2}\log(2|\mathbf{x}|) & |\mathbf{x}| > \frac{1}{2} \end{cases} \tag{2}$$

$$f_{ex}(\boldsymbol{\xi}) = 1 \tag{3}$$