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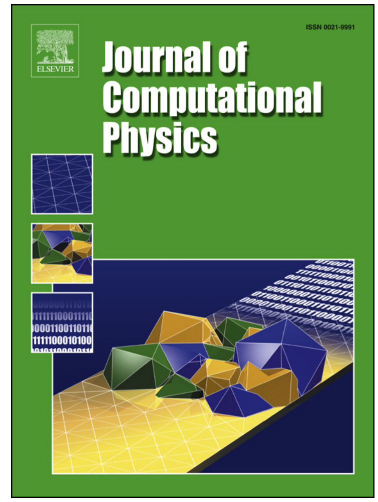

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Lyapunov exponents and adaptive mesh refinement for high-speed flows using a discontinuous Galerkin scheme

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Abstract

This paper proposes two important improvements to shock-capturing strategies using a discontinuous Galerkin scheme, namely, accurate shock identification via finite-time Lyapunov exponent (FTLE) operators and efficient shock treatment through a point-implicit discretization of a PDE-based artificial viscosity technique. The advocated approach is based on the FTLE operator, originally developed in the context of dynamical systems theory to identify certain types of coherent structures in a flow. We propose the application of FTLEs in the detection of shock waves and demonstrate the operator's ability to identify strong and weak shocks equally well. The detection algorithm is coupled with a mesh refinement procedure and applied to transonic and supersonic flows. While the proposed strategy can be used potentially with any numerical method, a high-order discontinuous Galerkin solver is used in this study. In this context, two artificial viscosity approaches are employed to regularize the solution near shocks: an element-wise constant viscosity technique and a PDE-based smooth viscosity model. As the latter approach is more sophisticated and preferable for complex problems, a point-implicit discretization in time is proposed to reduce the extra stiffness introduced by the PDE-based technique, making it more competitive in terms of computational cost.

Keywords: High-speed flows, Adaptive mesh refinement, Finite-time Lyapunov exponent, Discontinuous Galerkin, High-order methods

1. Introduction

The physical phenomena that take place in high-speed flows is of paramount importance specially in the aerospace context. For such flows, the presence of structures known as shock waves, through which the fluid properties experience an abrupt change, dictates the main features of the flow field. In typical aeronautical conditions, the thickness of a shock wave is so small (about 10^{-7}m) that it may be regarded as a discontinuity. This feature represents an extremely difficult challenge for numerical schemes which are designed to simulate such flows.

When a numerical scheme is used to solve a set of partial differential equations (PDEs) modeling the physical phenomena taking place, a discrete domain is chosen where an algebraic approximation to the PDEs is actually solved. Regardless of the numerical formulation adopted, the local error and hence the accuracy of the solution are functions of the local mesh size. In the vicinity of shocks, numerical schemes normally introduce unphysical wiggles, commonly related to the Gibbs' phenomenon. The methodologies developed to overcome this drawback usually reduce the local accuracy to first order in these regions, smearing the shock waves and increasing the associated numerical thickness. In such cases, only a local mesh refinement would diminish the numerical dissipation and provide a better shock resolution without the excessive computational cost that would follow from global mesh refinement. Thus, the use of local mesh refinement has become a very useful tool when dealing with high-speed flows and has been drawing the interest of the scientific community for a long time. In the past thirty years a large number of works addressed several refinement algorithms and methods to improve the solution convergence [1, 2, 3, 4]. Among those efforts, many used

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