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# A deep non-hydrostatic compressible atmospheric model on a Yin-Yang grid

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ABSTRACT

The singularity in the traditional spherical polar coordinate system at the poles is a major factor in the lack of scalability of atmospheric models on massively parallel machines. Overset grids such as the Yin-Yang grid introduced by Kageyama and Sato [1] offer a potential solution to this problem. In this paper a three-dimensional, compressible, non-hydrostatic atmospheric model is developed and tested on the Yin-Yang grid building on ideas previously developed by the authors on the solution of Elliptic boundary value problems and conservation on overset grids. Using several tests from the literature, it is shown that this model is highly stable (even with little off-centering), accurate, and highly efficient in terms of computational cost. The model also incorporates highly efficient and accurate approaches to achieve positivity, monotonicity and conservative transport, which are paramount requirements for any atmospheric model. The parallel scalability of this model, using in excess of 212 million unknowns and more than 6000 processors, is also discussed and shown to compare favourably with a highly optimised latitude–longitude model in terms of scalability and actual run times.



## 1. Introduction

Traditionally weather forecast and climate models have been developed using spherical polar coordinate systems. The singularity in this representation due to the poles is a major factor in the lack of scalability of atmospheric models on massively parallel machines. The development of the semi-implicit semi-Lagrangian schemes has mitigated many of the problems due to their robustness and ability to handle high Courant numbers stably. However the demand for simulations at higher and higher resolutions has put a strain on the scheme for two reasons. Firstly the condition number of the Helmholtz problem which arises through the semi-implicit treatment of acoustic and gravity waves increases rapidly with resolution and thus raises the number of iterations of the linear solvers dramatically as shown in, for example, [2]. Secondly the increased resolution near the pole gives rise to relatively high Courant numbers which generally need to be handled by communication-on-demand since the semi-Lagrangian departure points can originate on different processors in the parallel domain decomposition.

The use of overset grids is common in other fields of computational fluid dynamics such as aerodynamics, hydrodynamics and electromagnetics and fluid–structure interaction [3,4] where they are used as a means to simplify modelling of flows in complex geometries using simple rectangular overlapping grids (see [4] and references therein for more examples). Such a grid for the sphere, the Yin-Yang grid, was introduced by Kageyama and Sato [1] (see also [5]). It should also be

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mentioned that [6] and [7] employ the overset methodology in their treatment of the edges of the cubed sphere introduced by Sadourny [8] to reduce errors around these edges. From a computational point of view, the Yin-Yang Grid has many desirable properties, which are:

1. The spherical domain is decomposed into two identical rectangular grids without the problematic poles and with useful symmetry properties. This allows the equations in the usual spherical polar coordinate system to be employed in the numerical solution.
2. The grid is rectangular and orthogonal, which simplifies substantially the model's data structure. It is also suitable for finite difference/volume methods which permits accurate numerical solutions with minimum truncation errors.
3. The grid is free from singularities and special points, which simplifies the model/code's structure and enhances the code's computational efficiency and parallelism.
4. The grid spacing is quasi-uniform, which helps to produce a solution of similar quality over the whole domain and improves the load balancing on massively parallel machines.
5. Since the grid has a longitude–latitude structure, in principle all of the methods that have been proven to work effectively with such grids can be exploited.

Despite the numerous good properties of the Yin-Yang grid, the main concerns with composite grids are the overlap region and (i) how it affects wave propagation and (ii) how to effectively achieve conservation. In particular how these overlap regions can be coupled effectively to obtain a solution free from any significant grid-imprinting.

Recent works by the present authors have shown that these issues can be resolved effectively. In [2] it was shown how to effectively coupled the two panels to solve elliptic boundary value problems, arising from the semi-implicit treatment of the acoustic and gravity waves, without any significant impact of the overlapping region on the overall solution. It was also shown, in line with the results of [3], that provided the overlap regions are tightly couple and interpolated using a higher order scheme than that used in the interior then (i) the solution does not show any grid imprinting for smooth data and (ii) the condition number of matrix is significantly reduced.

More recently the problem of conservation and monotonicity was tackled in [9], which are both very important properties for the long time simulations involved in climate models as well as moisture and chemical (tracer) transport in limited area and global models where positivity and monotonicity are of paramount importance for any atmospheric model to produce realistic solutions.

Further evidence of the viability of the Yin-Yang approach can be found in [10,11]. In [10] the hydrostatic Canadian model was adapted to the Yin-Yang grid and it is shown that they can produce forecasts of similar quality to the longitude–latitude based model. Li et al. [11] also further demonstrated the viability of the approach with a full non-hydrostatic model.

In this paper, and building on previous works, a full three dimensional, deep and non-hydrostatic model on the Yin-Yang grid is presented and validated using several tests from the literature. The results show that this model is highly accurate and stable and compares very favourably with a highly optimised longitude–latitude based model, in terms of accuracy and computational efficiency. Using around 212 million degrees of freedom and more than 6000 processor it is shown that this, essentially unoptimised, model has good scalability and fast run times.

## 2. Basic geometry of the Yin-Yang grid

The Yin-Yang grid is essentially comprised of two longitude–latitude patches similar to that used in limited-area/regional weather forecasting. For completeness a short description will now be presented. Giving two Cartesian co-ordinate systems $(x, y, z)$ and $(x^*, y^*, z^*)$ two Latitude–Longitudinal $(\lambda, \phi)$ and $(\lambda^*, \phi^*)$ can be introduced such that

$$\begin{aligned} x &= \cos\lambda\cos\phi, & y &= \sin\lambda\cos\phi, & z &= \sin\phi, \\ x^* &= \cos\lambda^*\cos\phi^*, & y^* &= \sin\lambda^*\cos\phi^*, & z^* &= \sin\phi^*. \end{aligned} \tag{2.1}$$

The Yin-Yang grid is obtained by limiting $(\lambda, \lambda^*)$ to lie in the interval $[-\frac{3}{4}\pi - \delta, \frac{3}{4}\pi + \delta]$ and $(\phi, \phi^*)$ to $[-\frac{1}{4}\pi - \delta, \frac{1}{4}\pi + \delta]$ where $\delta$ is a small positive parameter and relating the two Cartesian systems by the orthogonal transformation

$$\begin{pmatrix} x^* \\ y^* \\ z^* \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}. \tag{2.2}$$

The resulting (overset) spherical grid is shown in Fig. 1. It follows trivially from (2.1) and (2.2) that given a local Longitudinal and Latitudinal coordinate $(\xi_1, \xi_2)$ on either panel then this point coordinates in the other panel can be obtained using Algorithm 1.

Denoting the orthonormal Euclidean basis on Yin by $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$ then the basis on Yang will be, via (2.2), $(\boldsymbol{i}^*, \boldsymbol{j}^*, \boldsymbol{k}^*) = (-\boldsymbol{i}, \boldsymbol{k}, \boldsymbol{j})$ so that the unit basis vectors on Yin and Yang are

$$\begin{aligned} \hat{\boldsymbol{\lambda}} &= -\sin\lambda\boldsymbol{i} + \cos\lambda\boldsymbol{j}, & \hat{\boldsymbol{\phi}} &= -[\cos\lambda\boldsymbol{i} + \sin\lambda\boldsymbol{j}]\sin\phi + \cos\phi\boldsymbol{k}, \\ \hat{\boldsymbol{\lambda}}^* &= \sin\lambda^*\boldsymbol{i} + \cos\lambda^*\boldsymbol{k}, & \hat{\boldsymbol{\phi}}^* &= [\cos\lambda^*\boldsymbol{i} - \sin\lambda^*\boldsymbol{k}]\sin\phi^* + \cos\phi^*\boldsymbol{j}, \end{aligned} \tag{2.3}$$