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Subdivision based isogeometric analysis technique for electric field integral equations for simply connected structures

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ABSTRACT

The analysis of electromagnetic scattering has long been performed on a discrete representation of the geometry. This representation is typically continuous but *not* differentiable. The need to define physical quantities on this geometric representation has led to development of sets of basis functions that need to satisfy constraints at the boundaries of the elements/tessellations (viz., continuity of normal or tangential components across element boundaries). For electromagnetics, these result in either curl/div-conforming basis sets. The geometric representation used for analysis is in stark contrast with that used for design, wherein the surface representation is higher order differentiable. Using this representation for *both* geometry and physics on geometry has several advantages, and is elucidated in Hughes et al. (2005) [7]. Until now, a bulk of the literature on isogeometric methods have been limited to solid mechanics, with some effort to create NURBS based basis functions for electromagnetic analysis. In this paper, we present the first complete isogeometry solution methodology for the electric field integral equation as applied to simply connected structures. This paper systematically proceeds through surface representation using subdivision, definition of vector basis functions on this surface, to fidelity in the solution of integral equations. We also present techniques to stabilize the solution at low frequencies, and impose a Calderón preconditioner. Several results presented serve to validate the proposed approach as well as demonstrate some of its capabilities.

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1. Introduction

Computational methods have become the mainstay of scientific investigation in numerous disciplines, and electromagnetics is no exception. Research in both integral equation and differential equation based methods has grown by leaps and bounds over the past few decades. This period has witnessed development of both higher order basis functions [1–5], and

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higher order representations of geometry (at least locally) [3,4,6] amongst many other equally important advances. These approaches have been applied to a wide range of realistic problems spanning several wavelengths.

However, despite advances in these areas, there is a fundamental disconnect between the geometry processing and analysis based on this geometry. Traditional analysis proceeds by defining a discrete representation of the geometry typically comprising piecewise continuous tessellations. Ironically, this discrete representation of the geometry is obtained using software or a computer aided design (CAD) tool that contains a higher order differentiable representation of the geometry. As eloquently elucidated in [7], the rationale for this disconnect can be attributed to the different periods in time that CAD tools and analysis tools were developed. As the latter is older, the computational foundation is older as well. As a result, one is left with awkward communication with the CAD software for refining and remeshing. This is especially true insofar as accuracy is concerned; lack of higher order continuity in geometry can cause artifacts if the underlying spaces for field representations are not properly defined. Indeed, the need to define div/curl conforming spaces on tessellations that are only C_0 led to development of novel basis sets that meet this criterion [8]. An alternate approach that has recently been espoused is isogeometric analysis (IGA). In this approach, the basis functions used to represent the geometry are the same as those used to represent the physics on this geometry. As a result, the features of geometry representation such as higher order continuity, adaptivity, etc., carry over to function representation as well. Since the appearance of this approach in 2005 [7], it has been applied to a number of applications that range from structure mechanics [9] to fluid–structure interactions (FSI) [10] to contact problems [11] to flow [12] to shell analysis [13,14] to acoustics [15] and electromagnetics [16]. In addition to analysis techniques, the power of IGA has been harnessed for design-through-analysis phase in several practical applications [17–19].

Next, we briefly review some of the existing methods. Most CAD tools use bi/tri-variate spline based patches/solids like those based upon Bezier, B-splines, and non-uniform rational B-splines (NURBS). As a result, these basis functions are the most often used as IGA basis, with the most popular one being NURBS. The latter choice is determined by the fact that NURBS is the industry standard for modern CAD systems. Properties such as non-negativity and the fact that it provides a partition of unity make it an excellent candidate for defining function spaces. Finite element methods based on NURBS basis functions that exhibit h- and p-adaptivity have been demonstrated [7]. Unfortunately, the challenge with using NURBS arises from the fact that the resulting shapes are topologically either a disk, a tube or a torus. As a result, stitching together these patches can result in surfaces that are not watertight. These complexities are exacerbated when the object being meshed is topologically complex or has multiple scales [20,21]. Two other geometry processing methodologies gaining popularity for handling shapes that are complex are T-splines and subdivision surface. The former, an extension to NURBS, can handle T-junctions and hence can greatly reduce the number of the control points in the control mesh. T-splines, especially analysis-ready T-splines, comprise a good candidate for constructing isogeometric analysis. More detailed work on T-splines and its application in IGA can be found in [21,22] and references therein. As opposed to T-splines, subdivision surfaces have played a significant role in the computer animation industry. Among its many advantages are the ease with which one can represent complex topologies, scalability, inherent multiresolution features, efficiency and ease of implementation. Furthermore, it converges to a smooth limit surface that is C^2 almost everywhere except at isolated points where it is C^1 continuous. There are several subdivision schemes; Loop, Catmull–Clark, Doo–Sabin to name a few. Generally speaking, all of the three of these schemes alluded above can be used to construct IGA method. To date, isogeometric analysis based on subdivision surface is less well studied. Some work on IGA based on Catmull–Clark can be found in [23], where IGA is used to solve PDEs defined on a surface.

While the literature on IGA for differential equations is reasonably widespread across multiple fields, IGA for integral equations is still at a nascent stage. As a result, it has recently become the focus of significant attention. Recently, two dimensional isogeometric boundary element method (IGBEM) was proposed [24] to study elastostatic problem with NURBS interpolating basis to represent the geometry, displacement and tractions. In [25], IGBEM based on unstructured T-splines was developed for a three dimensional linear elastostatic problem. This approach was extended to address IEs associated with hydrodynamic interactions [17]. Likewise, IGBEM methods have been developed to study acoustic scattering from rigid bodies [15] as well as two-dimensional electromagnetic analysis [26]. To our knowledge, IGA has not been used for solution to three dimensional IEs associated with vector electromagnetic fields, and this serves to motivate this paper.

The focus of this paper will be on the construction of a well formulated low-frequency stable IGA solver for the electric field integral equation (EFIE) that is based on subdivision surfaces specifically, the Loop subdivision scheme. As will be evident, the choice of Loop subdivision scheme is only incidental; the presented method can be applied to most subdivision surface descriptions. In developing a solver that is robust, several challenges need to be addressed; these range from definition of basis functions that correctly map the trace of fields on the surface to formulations that render the resulting system frequency stable to formulation of effective preconditioners. To set the stage for introduction of this formulation, we will assume that the surfaces are simply connected and have C^2 smoothness almost everywhere (extension to surfaces with sharp edges and corners is under development and will form the basis of a subsequent paper). As will be evident, the assumption of sufficient "local" smoothness permits significant freedom in terms of defining function spaces. Thus, the principal contribution of this work is five-fold: We will

- present construction of a basis to correctly represent the tangent trace fields on simply-connected surface,
- demonstrate convergence of EFIE-IGA solver for canonical geometries as well as present applications for complex targets,

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