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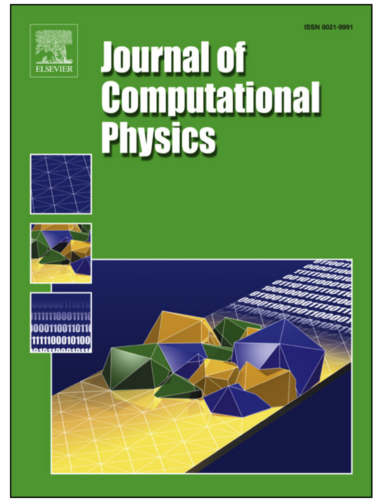

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Implementation of the Full Viscoresistive Magnetohydrodynamic Equations in a Nonlinear Finite Element Code

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Abstract

Numerical simulations form an indispensable tool to understand the behavior of a hot plasma that is created inside a tokamak for providing nuclear fusion energy. Various aspects of tokamak plasmas have been successfully studied through the reduced magnetohydrodynamic (MHD) model. The need for more complete modeling through the full MHD equations is addressed here. Our computational method is presented along with measures against possible problems regarding pollution, stability, and regularity.

The problem of ensuring continuity of solutions in the center of a polar grid is addressed in the context of a finite element discretization of the full MHD equations. A rigorous and generally applicable solution is proposed here.

Useful analytical test cases are devised to verify the correct implementation of the momentum and induction equation, the hyperdiffusive terms, and the accuracy with which highly anisotropic diffusion can be simulated. A striking observation is that highly anisotropic diffusion can be treated with the same order of accuracy as isotropic diffusion, even on non-aligned grids, as long as these grids are generated with sufficient care. This property is shown to be associated with our use of a magnetic vector potential to describe the magnetic field.

Several well-known instabilities are simulated to demonstrate the capabilities of the new method. The linear growth rate of an internal kink mode and a tearing mode are benchmarked against the results of a linear MHD code. The evolution of a tearing mode and the resulting magnetic islands are simulated well into the nonlinear regime. The results are compared with predictions from the reduced MHD model.

Finally, a simulation of a ballooning mode illustrates the possibility to use our method as an ideal MHD method without the need to add any physical dissipation.

Keywords: magnetohydrodynamics, finite element method, implicit time integration, magnetic vector potential, anisotropic diffusion, internal kink mode, tearing mode, ballooning mode

1. Introduction

The hot plasma in a tokamak knows many ways to escape the grip of the confining magnetic field. Most of these have been long understood and this understanding has led to a tremendous increase in the time a plasma can be kept in place. A first understanding of the development of instabilities in tokamak plasmas starts with an understanding of their linear phase. The theory of magnetohydrodynamics (MHD) has been successful in understanding many linear instabilities like kink, tearing, and ballooning modes. The kink mode is at the basis of what is called the sawtooth instability, named after the shape of the resulting magnetic signals. Tearing modes grow to form magnetic islands, while the ballooning mode plays an important role in the formation of Edge Localized Modes (ELMs). An accurate description of these phenomena typically requires the inclusion of effects outside of the MHD model, like the effects

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