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Multiscale modelling of evolving foams

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ABSTRACT

We present a set of multi-scale interlinked algorithms to model the dynamics of evolving foams. These algorithms couple the key effects of macroscopic bubble rearrangement, thin film drainage, and membrane rupture. For each of the mechanisms, we construct consistent and accurate algorithms, and couple them together to work across the wide range of space and time scales that occur in foam dynamics. These algorithms include second order finite difference projection methods for computing incompressible fluid flow on the macroscale, second order finite element methods to solve thin film drainage equations in the lamellae and Plateau borders, multiphase Voronoi Implicit Interface Methods to track interconnected membrane boundaries and capture topological changes, and Lagrangian particle methods for conservative liquid redistribution during rearrangement and rupture. We derive a full set of numerical approximations that are coupled via interface jump conditions and flux boundary conditions, and show convergence for the individual mechanisms. We demonstrate our approach by computing a variety of foam dynamics, including coupled evolution of three-dimensional bubble clusters attached to an anchored membrane and collapse of a foam cluster.

1. Introduction

1.1. Physics of foams

In this paper, we introduce, implement, and test a suite of numerical algorithms for computing solutions to a mathematical model of multi-scale foam dynamics.

Foams have a wide variety of applications in industry and materials design. For example, liquid foams, characterized by fluid-filled membranes separating gaseous regions, include soapy detergents and substances to separate out hydrophobic molecules; solid foams, formed by solidifying liquid foams, include lightweight materials such as metallic and plastic foams. Understanding the dynamics of foam evolution is a key step in controlling the structure and properties of foam-like materials. Deriving models to quantitatively predict foam evolution is challenging since the underlying physics takes place over vastly different time and space scales.

As a model of foam, consider common soap bubbles. A single isolated bubble consists of a thin membrane of fluid, known as the lamella, separating the inside gas from the outside. In a cluster of such bubbles, multiple lamellae meet at junctions known as Plateau borders, forming a network of interconnected thin-film membranes and borders. The dynamics of foam is intricate [57], and depends on a complex interaction between microscale fluid flow inside the lamellae and Plateau borders,

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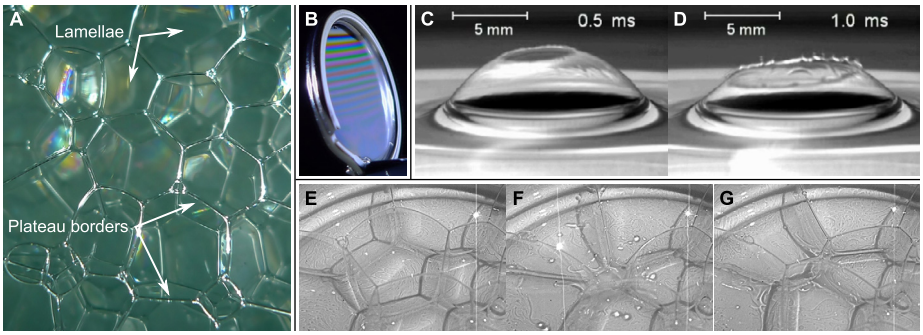

Fig. 1. (Reproduced from [44] and reprinted with permission from AAAS.) (A) A foam of soap bubbles made with common washing detergent. (B) Drainage and thin-film interference: A keyring suspended in soap solution makes a film, which then drains due to gravity. The subsequent variations in film thickness create interference patterns when lit with white light. (C, D) Rupture of a lamella; reproduced from [8] by permission from Macmillan Publishers Ltd., Nature, copyright 2010. (E, F, G) Rearrangement: A lamella (center of (E)) bursts, leading to macroscopic rearrangement of a foam.

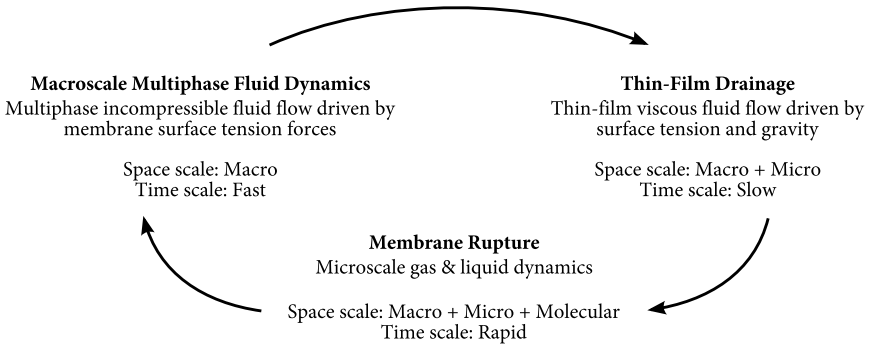

Fig. 2. Scale separated multiscale model of foam dynamics cycling between three stages: rearrangement, drainage, and rupture.

and the macroscale motion of the gas inside the bubbles. To illustrate, consider a foam whose macroscopic configuration appears to be in equilibrium, such as the foam in Fig. 1A. While seemingly stable, liquid inside the films drains over time, due to effects of gravity and surfactant. When one of the membranes becomes too thin, it ruptures and its liquid contents is redistributed, destroying the macroscopic equilibrium of the remaining membranes. Driven by macroscale gas dynamics and surface tension, these other membranes, as well as their film thicknesses, further change as they contort, stretch, and settle into a new equilibrium, setting the stage for continued fluid drainage.

These processes take place over six orders of magnitude in space and time. The liquid in the thin films, while only micrometers thick, drains over tens or hundreds of seconds (Fig. 1B) until a membrane ruptures (Figs. 1C, D). Membranes burst at hundreds of centimeters per second [8], causing macroscopic rearrangement of bubble geometry through surface and fluid forces occurring over less than a second (Figs. 1E, F, G).

This wide range of time and space scales leads to considerable algorithmic complexity. The resolution required to capture the microphysics associated with rupture becomes fundamentally impractical for resolving the large macroscale bubble evolution and rearrangement, even with today's advanced computing hardware and the most optimistic proposed future architectures.

1.2. A multiphysics scale separation approach

Fortunately, details at one space or time scale are not necessarily important at another scale. Exploiting this "scale-separation" can lead to models in which algorithms compute physics at different resolutions, and allow these different components to communicate across the scales.

In [44], we presented a scale-separated model for computing foam dynamics. The multi-scale model separates foam dynamics into a cycle of three distinct stages acting over different space and time scales (see Fig. 2): (i) a rearrangement phase, in which a foam out of macroscopic balance undergoes rearrangement due to surface tension and gas dynamics, leading to an equilibrium; (ii) a liquid drainage phase, in which the foam is essentially in macroscopic equilibrium, and the microscopic flow of liquid in the interconnected membranes is modelled until a lamella becomes too thin; and then (iii) a rupture phase, in which a lamella ruptures, sending the foam out of macroscopic balance, after which step (i) is invoked and the process repeated. Together, the dynamics of each phase affect the next, leading to a multi-scale model which captures the key effects of foam rearrangement, liquid drainage, and rupture.