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Mixed-hybrid and vertex-discontinuous-Galerkin finite element modeling of multiphase compositional flow on 3D unstructured grids

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ABSTRACT

Problems of interest in hydrogeology and hydrocarbon resources involve complex heterogeneous geological formations. Such domains are most accurately represented in reservoir simulations by unstructured computational grids. Finite element methods accurately describe flow on unstructured meshes with complex geometries, and their flexible formulation allows implementation on different grid types. In this work, we consider for the first time the challenging problem of fully compositional three-phase flow in 3D unstructured grids, discretized by any combination of tetrahedra, prisms, and hexahedra. We employ a mass conserving mixed hybrid finite element (MHFE) method to solve for the pressure and flux fields. The transport equations are approximated with a higher-order vertex-based discontinuous Galerkin (DG) discretization. We show that this approach outperforms a face-based implementation of the same polynomial order. These methods are well suited for heterogeneous and fractured reservoirs, because they provide globally continuous pressure and flux fields, while allowing for sharp discontinuities in compositions and saturations. The higher-order accuracy improves the modeling of strongly non-linear flow, such as gravitational and viscous fingering. We review the literature on unstructured reservoir simulation models, and present many examples that consider gravity depletion, water flooding, and gas injection in oil saturated reservoirs. We study convergence rates, mesh sensitivity, and demonstrate the wide applicability of our chosen finite element methods for challenging multiphase flow problems in geometrically complex subsurface media.

1. Introduction

Subsurface geological formations generally have complex geometries that require highly flexible meshing for accurate representation. Structured (or logically Cartesian) grids may not accurately describe many subsurface problems in hydrogeology and the recovery of hydrocarbon resources. They are also not well suited to model radial flow near wells, and results from commercial simulators may not converge in the near-well region [1].

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The most commonly used numerical method to model flow on structured grids is the finite difference (FD) approach, while finite volume (FV) methods are usually adopted for unstructured grids. In their lowest-order form, both assume element-wise constant scalar variables (such as saturations) and use a two-point flux approximation (TPFA) to compute vectors (fluxes) from (pressure) gradients between two points. It is well known that such lowest-order approximations suffer from excessive *numerical dispersion*, and *grid sensitivity*. The former can be reduced through 'brute force' by significantly refining the mesh, which is made more feasible by the development of massively parallelized simulators in the industry [2]. However, sufficient mesh refinement is often not feasible when modeling flow in field-scale hydrocarbon reservoirs or aquifers. Grid sensitivity cannot be resolved by mesh refinement. Specifically, it is well known that the TPFA may not converge unless the grid is K-orthogonal [3,4]. Recently, significant improvements have been made to the FV approach, for instance to accommodate the full permeability tensor [5–13] and fractures [14–16]. To improve FD flux computations on general grids and with tensor permeability, the multipoint flux approximation (MPFA) was introduced. In MPFA, fluxes are reconstructed from the pressures in multiple surrounding elements [17–21,6,22,23], similar to the stencil of a standard continuous Galerkin discretization. Several flavors of MPFA have been proposed since the original version [23,24,8,9,24–26]. MPFA has been compared to the Vertex Approximate Gradient scheme [27] and to BDM_1 space under numerical quadrature [24,25].

The last category of numerical flow models rely on finite elements (FE). FE are the method of choice in many disciplines in science and engineering that involve unstructured grids. The FE methods that we adopt in this work are motivated by two essential physical properties of flow through porous media: 1) pressures and fluxes are *continuous*, even across layers and fractures, while 2) fluid properties are often *discontinuous* across phase boundaries, fractures, and layers.

In light of the latter realization, we adopt the *discontinuous* Galerkin (DG) method for the mass transport update. DG is strictly mass conserving at the element level. In higher-order DG, compositions or saturations can be updated at all vertices or faces and the values can be discontinuous across faces. This is particularly useful in fractured or layered reservoirs. In this work, we employ a multi-linear DG approximation as a compromise between higher-order convergence versus the number of phase-split computations that have to be carried out at each degree-of-freedom. Many flavors of DG have been analyzed in terms of error estimates and convergence properties, and it is hard to do justice to the full scope of this work (the following papers provide an overview of pioneering and recents efforts in the analysis of DG methods: [28–52]).

We use a mixed-hybrid finite element (MHFE) to satisfy the second aforementioned physical property: that both pressure and flux fields are continuous everywhere. MHFE simultaneously and to the same order of accuracy solves for globally continuous pressure and flux fields [53,54,33,55–57]. The high accuracy in the velocity field in highly heterogeneous and/or anisotropic domains is the main attraction of the MHFE method. Computing the pressures on element faces is also convenient when modeling capillarity and fractured reservoirs. Unlike some FE methods, the MHFE-DG combination is strictly mass conserving at the element level.

A comparison between MFE and MPFA was presented in Matringe et al. [24] for single-phase incompressible flow without gravity. Hoteit and Firoozabadi [58] and [59] compared MHFE-DG to the traditional TPFA-FD approach in a commercial simulator, and to an equal-order MUSCL FV scheme, respectively. Both MPFA [60] and MHFE [61] flux approximations have been presented on unstructured 3D grids for two-phase incompressible flow. However, to the best of our knowledge, neither method has been investigated for unstructured 3D grids and (EOS-based) compositional and compressible multiphase flow with gravity, which is the subject of this work. We emphasize that the latter problem is governed by a different set of equations which involve the highly non-linear total compressibility and total partial molar volumes of multicomponent multiphase mixtures.

Based on this discussion, we adopt an implicit-pressure–explicit-composition (IMPEC) scheme with a higher-order DG explicit mass transport update and a MHFE implicit pressure and flux update. This MHFE-DG scheme was explored for single-phase compressible compositional flow in fractured media in Hoteit and Firoozabadi [58], and generalized to two-phase compositional flow in homogeneous [62] and fractured domains [63], all on structured 2D grids; and to two-phase immiscible and incompressible flow with capillarity on 3D unstructured grids in [64]. More recently, MHFE-DG has been applied to problems of increasing complexity and non-linearity: three-phase flow with an immiscible aqueous phase [65], three fully compositional multicomponent hydrocarbon phases or two hydrocarbon phases and a compositional aqueous phase modeled by the cubic-plus-association (CPA) equation-of-state (EOS) [66,67]. Fickian diffusion, three-phase capillarity, and discrete fractures were modeled in 3D in Moortgat et al. [68], Moortgat and Firoozabadi [69,70].

Our past studies of compositional multiphase flow have been restricted to structured grids. The objective of this work is to unleash the full potential of our FE methods by moving to unstructured grids and allowing for all types of commonly used elements: triangles and quadrilaterals in 2D, and hexahedra, prisms, and tetrahedra in 3D. One other improvement is that we consider vertex-based DG discretizations rather than face-based (which requires a different slope-limiter [53,64]). The superiority of this approach is demonstrated in one of the numerical examples.

We briefly summarize our mathematical fractional flow formulation in Section 2. In Section 3 we discuss the MHFE-DG implementation on unstructured grids. The numerical experiments in Section 4 are a main focus of this work. First, we model the recovery of hydrocarbon energy resources by three important processes (gravity depletion, water flooding, and compositional CO_2 injection) from a 3D reservoir discretized by 5 different structured and unstructured hexahedral, prismatic and tetrahedra grids. This example shows that we obtain the same results irrespective of grid-types for a wide range of multiphase processes that exhibit counter-current flow and phase behavior. Other sets of examples investigate grid sensitivity, anisotropic domains, and the convergence properties of the MHFE-DG method on structured and unstructured grids.