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A semi-implicit level set method for multiphase flows and fluid–structure interaction problems

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ABSTRACT

In this paper we present a novel semi-implicit time-discretization of the level set method introduced in [8] for fluid–structure interaction problems. The idea stems from a linear stability analysis derived on a simplified one-dimensional problem. The semi-implicit scheme relies on a simple filter operating as a pre-processing on the level set function. It applies to multiphase flows driven by surface tension as well as to fluid–structure interaction problems. The semi-implicit scheme avoids the stability constraints that explicit scheme need to satisfy and reduces significantly the computational cost. It is validated through comparisons with the original explicit scheme and refinement studies on two-dimensional benchmarks.

1. Introduction

Level set methods are classical methods to capture Lagrangian interfaces moving in complex flows [21]. In these methods the interface is implicitly given by the zero level of a function which is solution to an advection equation. Level set methods offer an alternative to interface capturing methods where interfaces are explicitly parametrized and followed along the flow. Compared to interface capturing methods one advantage of level set methods is their simplicity, since they only rely on the discretization of an advection equation, and their ability to follow topology changes of the interfaces. The accuracy of these methods, and in particular the fulfillment of conservation properties, rely on that of the discretization method used to solve the advection equation.

In Computational Fluid Dynamics, since the pioneering works [29,6] level set methods are mostly applied to compute multiphase flows. In that case the level set function is used to compute geometrical information on the interface, typically the local normal and curvature, which allows to express capillary forces. Surface advection can be coupled to the interface motion [32]. Several recent works, often combining level set methods with Volume-of-Fluid (VOF) methods, have in particular been devoted to improve the accuracy of the curvature evaluation [24,14].

Level set methods have been extended to account for elastic forces in fluid–structure interaction problems involving elastic membranes in three dimensions. Originally devised for membrane elasticity given by area changes in incompressible flows [7,8], these methods were further extended to the case of compressible flows [1] and to handle general 3D elasticity in a fully Eulerian framework [9]. They were implemented for biological applications in the context of three-dimensional

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finite-difference methods [17,5] or two-dimensional finite-element methods [25]. The case of elastic membranes with elasticity governed by shear and area change was recently studied in [19].

Whether based on front capturing Immersed Boundary Methods [22,23] or level set methods, stability issues often arise in the calculation of multiphase flows and fluid–structure interaction problems and impose some strong constraints on the size of the time-steps.

The stability of the Immersed Boundary Method is well known to be sensitive to the stiffness of the elastic force applied on the structure when it is discretized in an explicit fashion. In order to remove the time step restrictions, several works have been dedicated to the derivation and study of implicit schemes [31,18,28,27,20]. However these schemes seem to be of limited practical interest because of the large computational cost related to the iterative resolution of a strongly non-linear coupling at each time step. The definition of semi-implicit or approximate implicit schemes leads to more realistic methods (see [31,28,27,10,11] and [2] and the references therein for details about such schemes). The search for an unconditionally stable scheme for curvature forces in an immersed boundary framework was already addressed specifically for the calculation of surface tension [15,30].

There exist several ways to carry out the stability analysis of time-discretization schemes of fluid–structure coupling methods. Among them is the control of discrete energy conservation [2,20], which is based on the corresponding physical property of the continuous underlying models. Another way is to perform a linear analysis near an equilibrium state of the system. In [28,27] this approach enables the authors to show that the instability is in particular enhanced by the combination of a small fluid viscosity and a strong elastic force.

In the unpublished work [3] stability conditions on the time step for explicit and implicit discretization, schemes were derived for a linearized version of a simplified one-dimensional version of the level set model [8]. Despite the simplifications involved in this analysis, this *ad-hoc* model enabled us to exhibit relationships between the fluid viscosity, the elastic force, and the numerical parameters. The results allowed to recover already known results obtained in [4,12,13], depending on the physical parameters range. All these references indicate that, for stiff interfaces or high Reynolds numbers, explicit schemes require time step values of the order of $\lambda \Delta x^{3/2}$, where λ is the stiffness of the membrane, in fluid–structure interaction problems, or the surface tension in multiphase flows.

In the present work, we introduce a new semi-implicit scheme, which is linearly unconditionally stable and applies both to multiphase flows and to fluid–structure interaction problems. Our strategy consists of solving a diffusion equation to predict the interface position where curvature or elastic forces are computed.

An outline of this paper is as follows. In section 2 we recall the approach defined in [7,8] to derive level set methods for immersed membranes. In section 3 we summarize the stability analysis performed in [3]. In section 4 we derive our semi-implicit scheme and show its unconditional stability. Section 5 is devoted to numerical illustrations of the stability and accuracy of this semi-implicit scheme in two-dimensional flows, both for capillary flows and fluid–structure interactions. Finally, we draw in section 6 some conclusions and indicate directions for future work.

2. Level set methods for fluid–structure interaction

We recall here the derivation of Eulerian level set methods for the fluid–structure interaction resulting from an elastic membrane immersed in an incompressible fluid [7,8]. We consider a computational domain Ω in $\mathbb{R}^d$ with $d = 2$ or $d = 3$, filled with a viscous incompressible and homogeneous fluid of density $\rho > 0$ and viscosity $\mu > 0$, containing an elastic membrane $\Gamma_e(t)$. For simplicity here we assume that the membrane is massless, and external forces are neglected. We also assume that this membrane only reacts to area changes, but not to tangential shear. This is for instance the case for models of membrane cells considered in biophysics, such as phospholipidic bilayers.

We introduce a level set function ϕ initialized as the signed distance to the initial membrane $\Gamma_e(0)$, and such that:

$$\Gamma_e(t) = \{x \in \Omega, \phi(t,x) = 0\}, \ \forall t \in [0,T].$$

In the case when the tangential shear does not create a stress, it can be shown that the surface elastic energy for this membrane can be approached by the volume energy:

$$\mathcal{E}_\varepsilon(\phi) = \int_\Omega E_e(|\nabla\phi|)\frac{1}{\varepsilon}\zeta(\frac{\phi}{\varepsilon})dx \tag{1}$$

where $\varepsilon > 0$ is a small numerical parameter related to interface smoothing, and ζ a mollifying 1D function of total integral equal to 1. The elastic force that results from this energy is given by

$$F_\varepsilon[\phi] = \left\{\mathbb{P}_{\nabla\phi^\perp}(\nabla[E'(|\nabla\phi|)]) - E'(|\nabla\phi|)\kappa(\phi)\frac{\nabla\phi}{|\nabla\phi|}\right\}|\nabla\phi|\frac{1}{\varepsilon}\zeta(\frac{\phi}{\varepsilon}), \tag{2}$$

where $\mathbb{P}_{\nabla\phi^\perp} := \mathbb{I} - \frac{\nabla\phi\otimes\nabla\phi}{|\nabla\phi|^2}$ is the projector on the tangent plane, and ν_e and $\kappa(\phi)$ are respectively the stiffness coefficient and the mean curvature of $\Gamma_e(t)$. The Eulerian level set model then reads:

$$\begin{cases} \rho\,(\partial_t u + u\cdot\nabla u) + \nabla p - \mu\Delta u = F_\varepsilon \\ \operatorname{div} u = 0 \\ \partial_t\phi + u\cdot\nabla\phi = 0 \end{cases} \tag{3}$$