Journal of Computational Physics 314 (2016) 436–449

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

Third-order symplectic integration method with inverse time dispersion transform for long-term simulation

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ARTICLE INFO

Article history:
Received 29 November 2015
Received in revised form 11 March 2016
Accepted 12 March 2016
Available online 17 March 2016

Keywords:
Inverse time dispersion transform
Time-dispersion error
Pseudospectral method
Symplectic integration method
Long-term simulation

ABSTRACT

The symplectic integration method is popular in high-accuracy numerical simulations when discretizing temporal derivatives; however, it still suffers from time-dispersion error when the temporal interval is coarse, especially for long-term simulations and large-scale models. We employ the inverse time dispersion transform (ITDT) to the third-order symplectic integration method to reduce the time-dispersion error. First, we adopt the pseudospectral algorithm for the spatial discretization and the third-order symplectic integration method for the temporal discretization. Then, we apply the ITDT to eliminate time-dispersion error from the synthetic data. As a post-processing method, the ITDT can be easily cascaded in traditional numerical simulations. We implement the ITDT in one typical exiting third-order symplectic scheme and compare its performances with the performances of the conventional second-order scheme and the rapid expansion method. Theoretical analyses and numerical experiments show that the ITDT can significantly reduce the time-dispersion error, especially for long travel times. The implementation of the ITDT requires some additional computations on correcting the time-dispersion error, but it allows us to use the maximum temporal interval under stability conditions; thus, its final computational efficiency would be higher than that of the traditional symplectic integration method for long-term simulations. With the aid of the ITDT, we can obtain much more accurate simulation results but with a lower computational cost.

1. Introduction

Seismic modeling is an important foundation for exploration seismology, and synthetic seismograms are helpful for understanding wave phenomena in complex media. High-accuracy seismic modeling schemes are essential for high-resolution seismic interpretations. Seismic modeling methods can be classified into three main categories: direct methods, integral-equation methods, and ray-tracing methods [1]. The direct methods are the most popular since they can handle complicated wave phenomena associated with various structures well. The direct methods include three main kinds: finite-difference (FD) methods [2–4], pseudospectral methods [5–8], and finite-element methods [9]. Some hybrid methods, such as spectral-element methods [10] and finite-volume methods [11] have also been developed to achieve a much higher accuracy in numerical simulations.

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http://dx.doi.org/10.1016/j.jcp.2016.03.031

For direct seismic modeling methods, both space and time variables need to be discretized. A large sampling interval of discretization would bring severe numerical dispersion error; in contrast, a small sampling interval could reduce the numerical dispersion error but would greatly increase the computational cost. Therefore, it is necessary to develop high accuracy methods that can employ course grids in both temporal and spatial discretizations. Conventional FD schemes for spatial discretization lead to space dispersion error when computing the space derivatives of the wave equations. A typical exhibition of the space dispersion error is that the high frequency components of the wavefields do not propagate exactly with the expected velocity [12]. Numerous methods have been developed to eliminate space dispersion: the high-order FD schemes [12–16], optimized FD operators [17–26], the flux-corrected transport technique [27–29], the nearly analytical discrete methods [30,31], the stereo-modeling methods [32], and the pseudospectral methods [5–8].

Numerical discretization of temporal derivatives also introduces numerical artifacts, which are called time-dispersion error. The time-dispersion error is not as serious as the space dispersion error since it can be greatly reduced using a fairly small temporal interval. However, in the presence of long-term problems and large-scale models, a small temporal interval would tremendously increase the computational cost; for such cases, we have to use a large temporal interval to avoid overburdened computational cost. Unfortunately, a large temporal interval would introduce strong time-dispersion error as expected. For example, the second-order FD discretization have been widely used [33] due to its simplicity, but an extremely fine temporal interval is needed to minimize the time-dispersion error since its time-dispersion error is the most serious among all known methods.

To cope with the time-dispersion error, various methods have been developed, such as high-order time FD schemes [12,34–37], the rapid expansion method [38–41], low rank methods [42,43], Fourier finite-difference methods [44–46], correction methods based on filter and interpolation [47–50], and time dispersion transforms [51,52] as reviewed below.

Chen [34] presented three modeling schemes using high-order temporal discretization: the Lax–Wendroff method [1,12,53], the Nyström method [54–56], and the splitting method [57–59]. The splitting method is the same as the third-order symplectic integration method developed by Ruth [57]. Zhang et al. [35] and Zhang and Zhang [37] proposed a one-step extrapolation algorithm. This algorithm formulates the two-way wave equation as a first-order partial differential equation in time without suffering from numerical instability or time dispersion problems, which allows for a large temporal interval. However, their decomposition algorithm, optimized separable approximation [60], is expensive due to too many Fourier transforms [61].

Fomel et al. [42] approximated the wave extrapolation operators using the low rank approximation of a matrix operator in the mixed space-wavenumber domain. This method reduces computational cost by optimally selecting reference velocities and weights [61]. Song and Fomel [44] developed a related method, the Fourier finite-difference method, by cascading a Fourier transform operator and an FD operator to form a chain operator. The Fourier finite-difference method may have an advantage in efficiency because it uses only one pair of multidimensional forward and inverse fast Fourier transforms per temporal interval. However, it does not offer flexible controls on the approximation accuracy [43].

The rapid expansion method [39–41] incorporates Chebyshev polynomials during the approximation. This method employs many high orders for the temporal discretization (e.g., 8-order [40]) thus is suitable for large temporal intervals using concepts similar to the work presented by Tal-Ezer et al. [38]. Instead of using more terms in the expansion, Etgen and Brandsberg-Dahl [62] generalize the pseudospectral method to obtain pseudo-analytical solutions. They modify the Fourier transform of the Laplacian operator for the constant velocity model in an arbitrary number of space dimensions.

The time dispersion has proven to be independent of both the velocity model and the space dispersion model; furthermore, it is predictable since it only depends on the frequency, temporal interval, and propagation time [50]. Therefore, the time dispersion could be handled separately from space dispersion, without considering velocity variations. Liu et al. [49] formulated an explicit time evolution scheme in the time–space domain by introducing a cosine function approximation, in which optimum stencils and least-squares coefficients are introduced. Stork [48] proposed that the time-dispersion error is fully predictable and can be removed with careful filtering after FD modeling. Time dispersion is fixed by applying a time variable filter and interpolation. Dai et al. [50] showed more details and extended the mathematical analyses on Stork's work. Li et al. [47] showed two post-propagation filtering schemes based on the method proposed by Stork [48], and this type of correction method does not affect the computational efficiency much.

Wang and Xu [51,52] studied the time dispersion of pseudospectral methods and predicted it in theory. To remove the time-dispersion error, they proposed a time dispersion prediction algorithm (i.e. forward time dispersion transform) and correction algorithm (i.e. inverse time dispersion transform), which works for any order conventional time FD scheme. A relatively large temporal interval is allowed for wave propagation, which can greatly retain the computational efficiency while improving the accuracy [51].

For long-term simulations in large-scale seismic exploration and seismology [34,63–65], the conventional FD schemes on discretizing the temporal derivatives are not structure-preserving thus are extremely difficult to avoid error accumulations caused by the time dispersion. We can solve wave equations using symplectic integration methods (or the Hamiltonian dynamical systems) to reduce error accumulations [57,66,67]. On the other hand, a large temporal interval that is approaching the stability upper limit would greatly improve the computational efficiency. However, symplectic integration methods still suffer from error accumulations at fairly long travel times when using large temporal interval.

In this paper, we incorporate the inverse time dispersion transform (ITDT) [51,52] into the third-order symplectic integration method to reduce error accumulations when using a large temporal interval. We verify the superiority of our scheme by comparing it with the conventional second-order scheme and the rapid expansion method (8-order [40]). The