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Shallow-water sloshing in a moving vessel with variable cross-section and wetting–drying using an extension of George's well-balanced finite volume solver

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A B S T R A C T

A class of augmented approximate Riemann solvers due to George (2008) [12] is extended to solve the shallow-water equations in a moving vessel with variable bottom topography and variable cross-section with wetting and drying. A class of Roe-type upwind solvers for the system of balance laws is derived which respects the steady-state solutions. The numerical solutions of the new adapted augmented f-wave solvers are validated against the Roe-type solvers. The theory is extended to solve the shallow-water flows in *moving* vessels with arbitrary cross-section with influx–efflux boundary conditions motivated by the shallow-water sloshing in the ocean wave energy converter (WEC) proposed by Offshore Wave Energy Ltd. (OWEL) [1]. A fractional step approach is used to handle the time-dependent forcing functions. The numerical solutions are compared to an extended new Roe-type solver for the system of balance laws with a time-dependent source function. The shallow-water sloshing finite volume solver can be coupled to a Runge–Kutta integrator for the vessel motion.

1. Introduction

A class of high resolution wave-propagation finite volume methods is developed in [20] for multidimensional hyperbolic systems. These methods are based on solving Riemann problems for waves that define both first order updates to cell averages and also second order corrections which can be modified by limiter functions to obtain high resolution numerical solutions. The wave-propagation algorithms are modified in [5] for conservation laws and balance laws with spatially varying flux functions and are called f-wave-propagation methods. The main novel feature of the modified algorithms is to solve the Riemann problems by decomposition of the jump in the flux functions into waves propagating out from each grid cell interface instead of decomposition of the jump in cell averages. In [11,12] a class of augmented approximate Riemann solvers is developed for the single layer shallow water equations in the presence of a variable bottom surface using the f-wave-propagation algorithm. The solver is based on a decomposition of an augmented solution vector including the depth, momentum, momentum flux and the bottom surface. This solver is well-balanced, maintains depth non-negativity and extends to Riemann problems with an initial dry state.

The shallow-water equations over variable bottom topography and cross-section form a set of nonlinear hyperbolic conservation laws with geometric source terms due to the arbitrary cross-section constraining the flow. For a vessel with a

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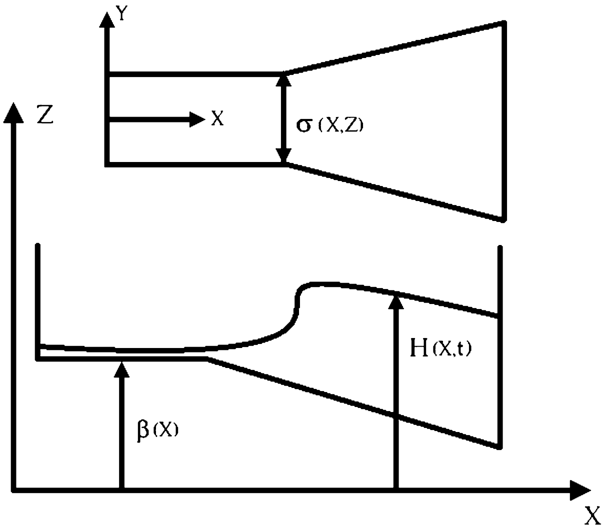

Fig. 1. Schematic of one-layer shallow-water sloshing over variable topography and cross-section in a stationary vessel.

symmetric rectangular cross-section with regard to a vertical plane passing through its longitudinal horizontal axis, the equations take the form

$$\begin{aligned}(h\sigma)_t + (hu\sigma)_x &= 0\,,\\ (hu\sigma)_t + \left(hu^2\sigma + \frac{1}{2}gh^2\sigma\right)_x &= \frac{1}{2}gh^2\sigma_x - gh\sigma\beta_x\,,\end{aligned} \tag{1}$$

where g is the gravitational constant, $h(x,t)$ is the fluid depth, $u(x,t)$ is the vertically averaged horizontal fluid velocity, $\beta(x)$ is the bottom topography, and $\sigma(x)$ is the breadth function such that the vessel walls are defined by the equations $y = \pm\frac{1}{2}\sigma(x)$. Here the subscripts denote partial derivatives. A schematic of the configuration of interest is shown in Fig. 1. The shallow-water equations (1) belong to the hyperbolic systems of the form

$$\mathrm{q}_t + f(\mathrm{q}, x)_x = \Psi(\mathrm{q}, x)\,, \tag{2}$$

where $\mathrm{q} \in \mathbb{R}^m$ is a vector of conserved quantities, $f(\mathrm{q}, x) \in \mathbb{R}^m$ is the vector of corresponding fluxes, and $\Psi(\mathrm{q}, x) \in \mathbb{R}^m$ is a vector of source terms. Non-trivial steady state solutions to (2) exist due to a balance of the flux gradient and the momentum source term due to variable topography and cross-section. Preserving the non-trivial steady states, or resolving small perturbations to them in the numerical solver when

$$f(\mathrm{q}, x)_x \approx \Psi(\mathrm{q}, x)\,, \tag{3}$$

and both terms are relatively large is a well-known difficulty which has received considerable studies, for instance see [30,10,5,6,22,23,12]. The other difficulty which arises in the numerical solution of the balance laws like (1) is the appearance and movement of a wet/dry front with vanishing or zero depth regions ahead or behind of the front [12]. Preserving depth non-negativity while maintaining mass conservation is particularly difficult with most standard Riemann solvers [12]. In [12] an augmented finite volume Riemann solver is developed for the shallow water equations over variable topography which preserves the steady states and maintains depth positivity in the Riemann solution along with satisfying other standard properties sought in Riemann solvers such as accurate capturing of wet/dry fronts and entropy requirements in the presence of large rarefaction waves.

This paper starts with the derivation of the shallow-water equations over a variable bottom topography and variable cross-section in moving coordinates with horizontal acceleration in §2. The interest in this paper is to develop an adapted version of the augmented f-wave finite volume solver of [12] for the shallow water equations (1) over variable cross-section and variable bottom topography which could handle Riemann problems with an initial dry state, and then modify the new solver to include a time-dependent source function to simulate shallow water sloshing with prescribed or coupled *surge* motion. The new augmented f-wave solver is validated against the Roe-type solver of [17] which is reviewed in §3. In §4 a new version of the Roe-type solver of [17] is derived for the shallow-water equations (1) with a time-dependent surge forcing function in the momentum equation (see equation (10)). The time-dependent augmented f-wave solver is then validated against the new Roe-type solver of §4. A review of the f-wave finite volume methods of [5,22] is given in §5. The detailed derivation of the new augmented f-wave finite volume solver for the shallow-water equations (1) is presented in §6. Section 7 is devoted to the numerical simulations and validations of the discussed solvers. Finally, some concluding remarks are given in §8.