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Proper orthogonal decomposition of flow-field in non-stationary geometry

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ABSTRACT

The current paper outlines a proper orthogonal decomposition (POD) methodology for a flow field in a domain with moving boundaries. In the standard POD approach the properties of the region of the domain, which alternatingly occupied by fluid and solid, are not defined. Here, prior to the decomposition, the domain with moving or deforming boundaries is mapped to a stationary domain using volume preserving mapping. This mapping was created by combining a transfinite interpolation and volume adjustment algorithm. The algorithm is based on an iterative solution of the Laplace equation with respect to the displacement potential of the grid points. Finally the method is demonstrated on CFD results of pitching and plunging ellipse in still fluid.

1. Introduction

The interaction of a fluid flow with moving or deforming boundaries is a key issue in the fields of aerodynamics and fluid mechanics; including wind energy [1–5], aero-elasticity [6,7], natural as well as bio-inspired flayers and swimmers [8–12]. In order to predict, simulate or control the effects of rigidly moving or deforming bodies on the flow and vice versa, dynamic models are highly desired, e.g. [13,14]. One common path in fluid mechanics is to use a Galerkin dynamic model [15,16] based on the modes obtained by proper orthogonal decomposition (POD) [17–19]. However, in the case of a domain with non-stationary boundaries, the standard POD approach does not meet the Galerkin system requirements [16,20], in the sense that the POD modes must be defined in a domain with stationary boundaries and must satisfy steady boundary conditions.

For certain cases with small boundary displacement, the deformation can be ignored and only forces acting on the fluid are considered [14,20]. For a single solid body kinematics, the Galerkin model can be constructed in body-fixed coordinates [21,22]. However, these methods are not applicable to domains with multiple moving boundaries or boundaries undergoing large amplitude motions or meaningful deformations. This paper is aimed to overcome the above mentioned limitations by introducing a method based on a Lagrangian–Eulerian approach [23]. The main idea enabling this method is to represent the flow state by the index of a Lagrangian grid point. This is achieved by creating a deforming domain which conserves volume and follows the kinematics of the immersed bodies on the interface between the solid and the fluid [20]. In order to achieve that, a volume preserving mapping is introduced. This mapping is a combination of transfinite (TF) interpolation

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[24] and volume adjustment algorithm. This approach provides, for some cases, a simpler alternative as compared to existing methods. As an example see [25–28], which in certain cases require more complex simulations or calculations.

The following sections outline the developed procedure, followed by the definition and derivation of the major stages of the algorithm, such as TF mapping and volume adjustment algorithm. Thereafter, the volume preserving mapping algorithm and the POD analysis are applied to a domain surrounding a pitching and plunging elliptic airfoil.

2. The procedure outline

The standard POD approach [15,16] is strictly limited to stationary fluid–solid interface with steady boundary conditions. In order to extend the existing POD methodology to flow states with non-stationary boundaries, the domain with moving boundaries should be mapped onto a domain with stationary boundaries, using volume preserving mapping, while keeping the domain outer boundary fixed with respect to the original boundary conditions, in both location and nature. The use of a general (transfinite interpolation (TF) [24] in this case) transformation combined with volume adjustment algorithm was currently adopted in order to create such a mapping. The steps comprising the procedure are outlined and described in some detail in the following paragraphs.

a) The stationary domain is generated using transfinite interpolation. The mesh which is defined within the unit square is mapped using Hermitian [24] interpolation onto the physical domain at its initial phase of motion as defined in Section 3. Here, each cell at the unit square is uniquely related to a cell in the physical domain and vice versa. This domain is then used in POD procedure.
b) Thereafter, the same mesh that has been defined on the unit square is mapped onto the other mesh phases of the motion cycle and all cell volumes are calculated.
c) The transfinite (TF) interpolation is not, in general, a volume preserving mapping, thus the initial mesh (which was defined in the unit square) should be adjusted in order to preserve volumes in the physical domain. In other words, a volume of a certain cell in a mesh corresponding to any subsequent phase should be made equal to that cell's volume in the zero-phase mesh. In order to perform such a mesh adjustment, a Poisson equation is defined and numerically solved as described in Section 4.
d) Following the volume adjustment, each cell at each phase of the cycle is related to the corresponding cell in the unit square and therefore to the cell in the physical domain at the initial phase of motion which was defined at step (a).
e) Next, the standard POD algorithm is applied to a set of velocity snapshots of the flow field perceived by their mapping onto the domain at its initial phase of the motion cycle.

The transfinite mapping and volume adjustment algorithm are described next, followed by an example of a POD analysis of the 2D flow field obtained from CFD simulation of pitching and plunging ellipse in still fluid at Reynolds number of 1000, based on the mean translational velocity.

3. Transfinite mapping

In order to generate the initial mesh, the use of transfinite mapping [24] is suggested. This transformation uses Hermitian interpolation in order to map a unit square defined in $(s,t) \in [0,1]$ onto the physical domain: $(\phi_i) = (x(\phi_i), y(\phi_i))$, where ϕ_i is the i-th phase of the boundary kinematics. The mapping is defined as follows:

$$\boldsymbol{X}(\phi_i) = \left(2t^3 - 3t^2 + 1\right) \cdot \boldsymbol{F}_1(s,\phi_i) + \left(-2t^3 + 3t^2\right) \cdot \boldsymbol{F}_2(s) + + \left(t^3 - 2t^2 + t\right) \cdot \boldsymbol{G}_1(s,\phi_i) + \left(t^3 - t^2\right) \cdot \boldsymbol{G}_2(s) \tag{1}$$

Here: $\boldsymbol{F}_1 = (F_{1x}(s,\phi_i), F_{1y}(s,\phi_i))$ and $\boldsymbol{F}_2 = (F_{2x}(s), F_{2y}(s))$ are the inner (moving) and outer (stationary) boundaries; while $\boldsymbol{G}_1 = (G_{1x}(s,\phi_i), G_{1y}(s,\phi_i))$ and $\boldsymbol{G}_2 = (G_{2x}(s), G_{2y}(s))$, are the directions (or partial derivatives) of the interpolation polynomials at the boundaries. Those function provide additional quality control of the generated mesh. Note, that in many cases, multiple bodies can also be defined using a single contour $\boldsymbol{F}_1(s,\phi_i)$ by connecting those bodies by a zero-volume "wire" of varying length.

4. Volume adjustment algorithm

Following the transfinite mapping, each point in the unit square in the (s,t) plane can be uniquely mapped onto each point in each phase of the motion cycle. However, the TF mapping is not volume preserving, thus the kinematics of those points do not satisfy the continuity equation. Therefore, the location of each point should be adjusted. For this adjustment, the initial mesh in the unit square in the (s,t) plane is modified in an iterative manner to satisfy the following equation:

$$\Delta^{(s,t)} \psi_i = 1 - \frac{V^{(x,y)}(\phi_0)}{V^{(x,y)}(\phi_i)} \tag{2}$$

Here $\Delta^{(s,t)} = \frac{\partial^2}{\partial s^2} + \frac{\partial^2}{\partial t^2}$ is the Laplacian operator and ψ_i is the displacement potential in the (s,t) plane related to the i-th phase. Therefore, the displacements are $\delta s = \frac{\partial \psi_i}{\partial s}; \delta t = \frac{\partial \psi_i}{\partial t}$. In the above, $V^{(x,y)}(\phi_i)$ is the local volume distribution in the (x,y) plane of the i-th snapshot.