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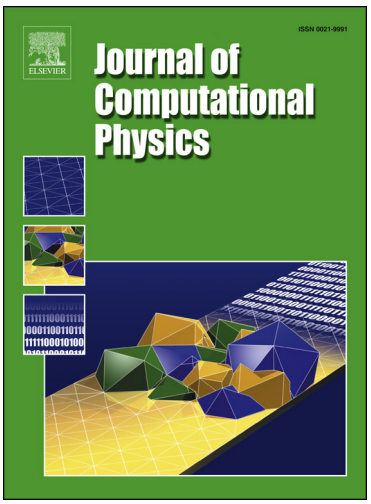

A fast finite volume method for conservative space-fractional diffusion equations in convex domains

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A fast finite volume method for conservative space-fractional diffusion equations in convex domains

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Abstract

We develop a fast finite volume method for variable-coefficient, conservative space-fractional diffusion equations in convex domains via a volume-penalization approach. The method has an optimal storage and an almost linear computational complexity. The method retains second-order accuracy without requiring a Richardson extrapolation. Numerical results are presented to show the utility of the method.

Keywords: anomalous diffusion, circulant matrix, conjugate gradient squared method, fast Fourier transform, space-fractional diffusion equation, Toeplitz matrix, volume penalization

1. Introduction

Fractional partial differential equations (FPDEs) provide powerful alternatives to integer-order PDEs for modeling challenging phenomena such as anomalous transport, long-range interactions, and nonlocal dynamics [3, 8, 21, 23]. However, FPDEs involve complex integral operators with singular kernels. Consequently, their numerical discretizations tend to generate dense stiffness matrices, for which traditionally used direct solvers [16, 17, 19, 24, 27] require $O(N^2)$ memory and $O(N^3)$ computations for a problem of size N. The significantly increased computational complexity and memory requirement render realistic multidimensional FPDE modeling and simulations computationally intractable.

Extensive effort has been made to develop efficient and accurate numerical methods for multidimensional FPDEs. Meerschaert et al [19, 27] devel-