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A flux-splitting method for hyperbolic-equation system of magnetized electron fluids in quasi-neutral plasmas

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A B S T R A C T

A flux-splitting method is proposed for the hyperbolic-equation system (HES) of magnetized electron fluids in quasi-neutral plasmas. The numerical fluxes are split into four categories, which are computed by using an upwind method which incorporates a flux-vector splitting (FVS) and advection upstream splitting method (AUSM). The method is applied to a test calculation condition of uniformly distributed and angled magnetic lines of force. All of the pseudo-time advancement terms converge monotonically and the conservation laws are strictly satisfied in the steady state. The calculation results are compared with those computed by using the elliptic–parabolic-equation system (EPES) approach using a magnetic-field-aligned mesh (MFAM). Both qualitative and quantitative comparisons yield good agreements of results, indicating that the HES approach with the flux-splitting method attains a high computational accuracy.

1. Introduction

Quasi-neutral plasma flows appear in many practical applications such as space propulsion, astrophysics, and nuclear fusion [1–4]. In plasma simulations of these applications, the model of lightweight electrons is usually considered separately from the models of ions and neutral particles. When considering electron fluids in quasi-neutral plasmas, the space potential is solved by using the electron conservation equations, rather than by using Gauss's law [5]. Because of the electrical neutrality, the electron number density distribution is given by the ion number density distribution. Therefore, the electron velocity, electron temperature, and space potential are calculated by using the conservation equations for mass, momenta, and energy of electrons.

The conventional approaches utilize an elliptic equation and a parabolic equation for solving for the space potential and electron temperature [6,7]. In what follows, this approach will be referred to as the elliptic–parabolic-equation system (EPES) approach. However, in the case of magnetized electrons, the EPES becomes an anisotropic diffusion problem, owing to magnetic confinement. Computation of this system becomes difficult owing to: 1) the anisotropy stemming from the large difference between diffusion coefficients in different directions and 2) the instability caused by cross-diffusion terms. The cross-diffusion terms are especially difficult to handle because they cause the failure of the diagonal dominance of the coefficient matrix. One approach toward avoiding the issue of cross-diffusion terms is to utilize a magnetic-field-aligned mesh (MFAM) [8]. By precisely aligning the computational mesh with the magnetic lines of force, the cross-diffusion terms

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can be neglected because they stem from the angle between the magnetic lines of force and the mesh. However, using an MFAM makes it impossible to use a structured mesh for the body-fitted coordinate system and complicates the evaluation of fluxes on the mesh boundaries. Furthermore, once the magnetic field induced by the plasma current is solved, one needs to reconstruct the mesh with varying magnetic lines of force. Thus, a practical application of the MFAM is associated with cumbersome coding and implementation steps.

Recently, a novel approach to solving anisotropic diffusion equations has been proposed, which uses a hyperbolic-equation system (HES) [9]. The key idea in this approach is to construct a hyperbolic system by introducing pseudo-time advancement terms. This approach avoids the aforementioned difficulties related to the anisotropy and cross-diffusion terms. It was confirmed that an anisotropic diffusion problem of space potential was robustly computed by using the HES approach. It was proved that the advantage of the HES approach compared with the MFAM-based approach was that it could use a simple structured mesh without increasing the computational cost.

Although the HES approach demonstrated advantages for calculating magnetized electron flows, there remain two issues constraining the applicability of this approach. First, the HES approach was proposed only for mass and momentum conservation equations. To simulate plasma devices utilizing the heating of electrons for the plasma generation, the HES must include the energy conservation equation for deriving the electron temperature. Another issue is the presence of large numerical viscosity. It was reported that the HES approach had a large numerical viscosity arising from the discretization of cross-diffusion terms [9]. In light of these issues, the purpose of this paper was to extend the HES to include all conservation laws, and to find a robust method for computing the system of equations. In addition, a high-order spatial accuracy method was used with the HES approach for reducing the numerical viscosity. As a criterion of computational accuracy, we checked whether the HES approach achieves the same level of computational accuracy as the MFAM-based EPES approach.

2. Hyperbolic system of conservation laws for electron fluid

2.1. Hyperbolic-system formulation of energy conservation equation

The fundamental equations are the two-dimensional equations of electron mass, momentum, and energy conservation in quasi-neutral plasma. Assuming quasi-neutrality, the electron number density is equal to the ion number density. Thus, the electron number density is treated as a given distribution. Also, the inertia of the electron fluid is neglected in the momentum conservation equation, because of frequent collisions. The detailed processes to derive the fundamental equations for the mass and momentum conservations can be found in Ref. [9]. Thus, this section focuses on the equations for the energy conservation.

Energy conservation is formulated as follows:

$$\frac{\partial}{\partial t}\left(\frac{3}{2}en_{\mathrm{e}}T_{\mathrm{e}}\right)+\nabla\cdot\left(\frac{5}{2}en_{\mathrm{e}}T_{\mathrm{e}}\vec{u}_{\mathrm{e}}-\frac{5}{2}en_{\mathrm{e}}T_{\mathrm{e}}\,[\mu]\,\nabla T_{\mathrm{e}}\right)=en_{\mathrm{e}}\vec{u}_{\mathrm{e}}\cdot\nabla\phi-\alpha e\varepsilon_{\mathrm{ion}}n_{\mathrm{e}}\nu_{\mathrm{ion}}, \tag{1}$$

where e, n_{e}, u_{e}, T_{e}, ϕ, ν_{ion}, and $\varepsilon_{\mathrm{ion}}$ are the elemental charge, electron number density, electron velocity, electron temperature, space potential, ionization collision frequency, and first ionization energy of the gas species used for the plasma generation, respectively. α is a coefficient to handle ionization, excitation, and radiation with a single term, and it is experimentally determined as a function of the electron temperature [10]. It is assumed that the conservation of energy also achieves a steady state on the ion time scale. The electron mobility tensor $[\mu]$ on a computational mesh is derived as follows:

$$[\mu]=\begin{bmatrix}\mu_{\mathrm{x}} & \mu_{\mathrm{c}}\\ \mu_{\mathrm{c}} & \mu_{\mathrm{y}}\end{bmatrix}=\Theta^{-1}\,[\mu]_{\mathrm{mag}}\,\Theta, \tag{2}$$

where

$$[\mu]_{\mathrm{mag}}=\begin{bmatrix}\mu_{||} & \\ & \mu_{\perp}\end{bmatrix}=\begin{bmatrix}\frac{e}{m_{\mathrm{e}}\nu_{\mathrm{col}}} & \\ & \frac{\mu_{||}}{1+(\mu_{||}|B|)^2}\end{bmatrix},\qquad \Theta=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}. \tag{3}$$

Here, m_{e}, ν_{col}, and B are electron mass, total collision frequency, and magnetic flux density, respectively. The subscripts $||$ and $\perp$ denote parallel and perpendicular directions of magnetic lines of force, respectively. Θ is the rotation matrix, with the angle between the magnetic lines of force and the computational mesh.

In quasi-neutral plasmas, the plasma approximation is assumed and the space potential is calculated from the conservation equations for the electrons [5]. In conventional approaches, the mass and momentum conservation equations are integrated into an elliptic equation [6,7]. However, this equation becomes an anisotropic diffusion equation, and it is difficult to maintain stability while computing this equation because the cross-diffusion terms cause failure of the diagonal dominance of the coefficient matrix [9]. Alternatively, the HES approach using pseudo-time advancement terms is considered. The HES which consists of the mass and momentum conservation equations has been proposed in Ref. [9].