Journal of Computational Physics 310 (2016) 252–284

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

Coupled acoustic response of two-dimensional bounded and unbounded domains using doubly-asymptotic open boundaries

C. Birk [a,*], L. Liu [b], Ch. Song [b]

[a] Institute of Structural Analysis of Plates and Shells, University of Duisburg-Essen, 45141 Essen, Germany
[b] School of Civil and Environmental Engineering, University of New South Wales, Sydney, NSW 2052, Australia

ARTICLE INFO

Article history:
Received 7 June 2015
Received in revised form 12 December 2015
Accepted 14 December 2015
Available online 12 January 2016

Keywords:
High-order open boundary
Doubly asymptotic
Circular cavity
Exterior acoustics
Continued-fraction expansion
Scaled boundary finite element method

ABSTRACT

A high-order doubly-asymptotic open boundary for modelling scalar wave propagation in two-dimensional unbounded media is presented. The proposed method is capable of handling domains with arbitrary geometry by using a circular boundary to divide these into near field and far field. The original doubly-asymptotic continued-fraction approach for the far field is improved by introducing additional factor coefficients. Additionally, low-order modes are approximated by singly-asymptotic expansions only to increase the robustness of the formulation. The scaled boundary finite element method is employed to model wave propagation in the near field. Here, the frequency-dependent impedance of bounded subdomains is also expanded into a series of continued fractions. Only three to four terms per wavelength are required to obtain accurate results. The continued-fraction solutions for the bounded domain and the proposed high-order doubly-asymptotic open boundary are expressed in the time-domain as coupled ordinary differential equations, which can be solved by standard time-stepping schemes. Numerical examples are presented to demonstrate the accuracy and robustness of the proposed method, as well as its advantage over existing singly-asymptotic open boundaries.

1. Introduction

The propagation of acoustic waves in unbounded domains, including sound wave radiation and scattering by obstacles, is of importance in many practical applications such as sonar, crack detection, acoustic optimization of vehicles and the seismic design of dam–reservoir systems. The numerical modelling of these types of problems is challenging due to the requirement to satisfy the radiation condition at infinity. Moreover, the modelling of waves in bounded domains poses its own problems with regards to fine mesh requirements when high wavenumbers are involved.

With respect to accurately and efficiently representing unbounded domains, many different numerical approaches have been proposed. The most popular methods used in exterior acoustics include the boundary element method [1], infinite elements [2,3], perfectly matched layers [4–6] and absorbing boundaries [7,8]. A detailed review and comparison with respect to advantages and limitations is given in Reference [9]. Each of these methods exhibits its distinct benefits and shortcomings. Ideally, a numerical method for waves in infinite media should be local in time, applicable to transient

* Corresponding author. Tel.: +49 201 183 2667; fax: +49 201 183 3965.
E-mail addresses: carolin.birk@uni-due.de (C. Birk), lei.liu@unsw.edu.au (L. Liu), c.song@unsw.edu.au (Ch. Song).

http://dx.doi.org/10.1016/j.jcp.2015.12.029
0021-9991/

analysis and arbitrary geometry, accurate, stable, easy to implement and to couple to a computational model of the near field.

Local absorbing boundary conditions are particularly attractive in terms of efficiency and ease of implementation, but classical low-order formulations [10] have been shown to lack accuracy in a number of cases. High-order absorbing boundary conditions have been developed in order to improve accuracy, well-known early examples include [11–13]. Although the order of these formulations can, in theory, be increased to any desired value, thereby leading to improved accuracy, the associated numerical algorithms may become unstable for orders higher than two. In addition, the implementation in the time-domain is not straightforward, due to the high-order derivatives. This problem can be overcome by replacing the high-order derivatives with auxiliary variables, as has first been proposed in Reference [14] and further developed in References [15–18].

Most of the proposed high-order absorbing boundaries are only singly asymptotic at the high-frequency limit. That is, they are suitable for radiative fields where all of the field energy propagates to infinity. This is not the case in a layered system, where evanescent modes exist. Even if all modes are propagating, the rate of convergence of high-order singly-asymptotic open boundaries may be low. For a circular cavity embedded in a full plane, the modal stiffness coefficient approaches that of a horizontal layer with increasing mode number λ [19]. The slow convergence can be overcome by using a doubly-asymptotic approximation (DAA), which is accurate not only at the high-frequency limit, but also at statics. The development of the DAA is documented in References [20–25]. The highest order of the DAA reported in the literature is three [25].

In Reference [19] a high-order doubly-asymptotic open boundary was proposed specifically for the modal equations of scalar waves. This formulation has been shown to accurately model evanescent waves and long-time responses. Compared to singly-asymptotic open boundaries, it leads to a significant gain in accuracy at no additional cost. The method proposed in Reference [19] is based on expanding the modal stiffness coefficient into a series of continued fractions. The continued-fraction expansion is expressed as a system of first-order differential equations in the time domain by introducing auxiliary variables. Thus, well-established time-stepping schemes are directly applicable. In Reference [26], however, it has been demonstrated that the algorithm proposed in [19] can become ill-conditioned for wavenumbers λ close to $i + 0.5$, where i is an integer. Therefore, an improvement is proposed in this paper to increase the numerical robustness of the solution procedure. This is achieved by introducing additional factor coefficients in the derivation of the doubly-asymptotic continued-fraction solution. As a result, the denominators of certain continued-fraction coefficients turn into only sign functions, whereby singularities are avoided. In addition, it is also proposed to use singly-asymptotic approximations for the low-order modes only and to combine these with doubly-asymptotic approximations of all other modes. This contributes to improving robustness and reducing the computational cost.

At the same time, the approach presented in Reference [19] for modal equations is extended to the two-dimensional case in this paper. To this end, it is combined with a scaled boundary finite element approach. The scaled boundary finite element method is a semi-analytical technique that is particularly suitable for modelling waves in unbounded domains and for representing singularities. It has also been used in the context of dynamic dam–reservoir interaction [27]. The development of the method is documented in References [28–33].

A two-dimensional acoustic domain of arbitrary shape can be divided into a near field region containing any irregular geometrical features and a far field region by introducing a circular boundary. The near field can be further divided into bounded scaled boundary finite element subdomains, which can be regarded as superelements. The far field region is represented by the proposed high-order doubly-asymptotic open boundary. The scaled boundary finite element equations of the unbounded domain with circular boundary are decoupled in this approach. The resulting modal scaled boundary finite element equations can be cast in terms of a frequency-dependent modal impedance coefficient and the solutions can be sought recursively by expanding this coefficient into a series of continued fractions and satisfying the scaled boundary finite element equation at both high and low frequency limit. Via introducing auxiliary variables and superimposing the contributions of individual modes, the continued-fraction expansion is transformed into a temporally local open boundary condition in the time-domain [19]. The resulting system of first-order differential equations to represent the unbounded domain is easily coupled to the standard equations of motion representing the interior via the nodal flux vector on the circular boundary.

This paper is organized as follows. The scaled boundary finite element approach for two-dimensional linear acoustics is outlined in Section 2. The improved doubly-asymptotic open boundary is derived in Section 3. The coupling of the bounded domain scaled boundary finite element model and the high-order doubly-asymptotic open boundary is addressed in Section 4. Numerical examples are presented in Section 5 to illustrate the improved numerical robustness, accuracy and efficiency of the proposed approach.

2. Scaled boundary finite element approach for two-dimensional linear acoustics

Wave propagation in a linear acoustic medium is governed by the scalar wave equation,

$$\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0. \tag{1}$$