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A flexible genuinely nonlinear approach for nonlinear wave propagation, breaking and run-up

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A B S T R A C T

In this paper we evaluate hybrid strategies for the solution of the Green–Naghdi system of equations for the simulation of fully nonlinear and weakly dispersive free surface waves. We consider a two step solution procedure composed of: a first step where the non-hydrostatic source term is recovered by inverting the elliptic coercive operator associated to the dispersive effects; a second step which involves the solution of the hyperbolic shallow water system with the source term, computed in the previous phase, which accounts for the non-hydrostatic effects. Appropriate numerical methods, that can be also generalized on arbitrary unstructured meshes, are used to discretize the two stages: the standard C^0 Galerkin finite element method for the elliptic phase; either third order Finite Volume or third order stabilized Finite Element method for the hyperbolic phase. The discrete dispersion properties of the fully coupled schemes obtained are studied, showing accuracy close to or better than that of a fourth order finite difference method. The hybrid approach of locally reverting to the nonlinear shallow water equations is used to recover energy dissipation in breaking regions. To this scope we evaluate two strategies: simply neglecting the non-hydrostatic contribution in the hyperbolic phase; imposing a tighter coupling of the two phases, with a wave breaking indicator embedded in the elliptic phase to smoothly turn off the dispersive effects. The discrete models obtained are thoroughly tested on benchmarks involving wave dispersion, breaking and run-up, showing a very promising potential for the simulation of complex near shore wave physics in terms of accuracy and robustness.

1. Introduction

The accurate mathematical and numerical simulations of water wave propagation in near-shore regions have received considerable attention in the last decades, since they have largely replaced laboratory experiments in the coastal engineering community. Significant efforts have been made in the development of depth averaged models or in the improvement of the existing ones, in order to give accurate description of the nonlinear and non-hydrostatic propagation over complex bathymetries.

The use of asymptotic depth averaged models on this task is quite common, since they lead to numerical models that are of practical use in design compared to the ones produced by more complicated mathematical models like the Euler equations. One of the most known depth averaged models, widely used, is the non-linear shallow water equations (NLSW). This set of equations is capable of providing a good description of the non-linear transformation of the waves, including also

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wave breaking but they lack on describing all the dispersive effects that play an important role on deeper waters and on wave shoaling. As to take into account the dispersion effects, the use of asymptotic depth averaged Boussinesq and enhanced Boussinesq [54,48,46,10,80] type models is quite common. A review on the history and all the fundamental aspects of the Boussinesq-type models can be found in [15].

In the last decades a system of equations, produced by the Euler equations, has gained a lot of attention. Green and Naghdi [31] derived a fully non-linear and weakly dispersive set of equations for an uneven bottom, which represents a two dimensional extension of the Serre equations [66]. They are known as Serre or Green–Naghdi (GN), or fully non-linear Boussinesq equations. The range of validity of the model may vary as much as far the non-linearity parameter ϵ (defined as the ratio of wave amplitude to water depth A/h_0) is concerned, but it requires the shallowness parameter μ (defined as the water depth to wavelength ratio h_0/L) to be small (less than one). The GN model has been fully justified mathematically [40] in the sense that the error between the solutions of the GN system and the Euler equations is small and of size $O(\mu^2)$. We refer to [41,12] for more details.

From the numerical point of view the GN equations have been discretized using different numerical techniques like Finite Differences (FD), Finite Elements (FE) and Finite Volume (FV) approaches. We refer to [3,22,23,20,12,42,44,50,27] among others. For example, in [22,23] the authors derive a higher order FV scheme in one dimension. In [20,12] a hybrid FV/FD splitting approach is used, while [42] follows the same idea for the solution of a new class of two-dimensional GN equations on structured meshes. In [44] a coupled Discontinuous Galerkin and Continuous Galerkin method is developed in one dimension but using only flat bottom topographies. Most of them are also really hard to extend in two dimensions. Up to now and to the authors knowledge there is no work that involves the solution of the later equations in 2D unstructured meshes.

Like all the Boussinesq-type models, GN equations cannot reproduce the energy dissipation that take place when a wave is breaking, producing satisfactory results only outside the breaking region. For this reason the numerical model must be incorporated with a wave breaking mechanism as to handle broken waves. Several approaches have been developed among the years. An extensive review of the existing wave breaking techniques can be found in [37].

In this work, our first aim is to evaluate a strategy that can be easily generalized on arbitrary unstructured meshes, and in the multidimensional case, for the solution of fully nonlinear, weakly dispersive free surface waves. For this reason we consider the hybrid approach, used e.g. in [12] and [37], simulating the propagation and shoaling by means of the Green–Naghdi partial differential equations (PDEs), while locally reverting to the non-linear shallow water equations to model energy dissipation in breaking regions. Starting from the form of the Green–Naghdi equations proposed in [12] and [20], we consider a two step solution procedure: an elliptic phase in which a source term is computed by inverting the coercive operator associated to the dispersive effects; an hyperbolic phase in which the flow variables are evolved by solving the nonlinear shallow water equations, with all the non-hydrostatic effects accounted for by the source computed in the elliptic phase. For the numerical discretization of these two steps we consider methods which can be easily generalized on arbitrary unstructured meshes in the multidimensional case. In particular, we focus on the use of a standard C^0 Galerkin finite element method for the elliptic phase, while high order finite volume (FV) and stabilized finite element (FE) methods are used independently in the hyperbolic phase. The discrete dispersion properties of the fully coupled methods obtained are also studied, showing phase accuracy very close to that of a fourth order finite difference method.

In addition, we will exploit the two step solution procedure to obtain a robust embedding of wave breaking. We evaluate two strategies: one, based on simply neglecting the non-hydrostatic contribution in the hyperbolic phase; the second, involving a tighter coupling of the two phases, with a wave breaking indicator embedded in the elliptic phase to smoothly turn off the dispersive effects. The discrete models obtained are thoroughly tested on benchmarks involving wave dispersion, breaking and run-up, showing a very promising potential for the simulation of complex near shore wave physics.

The paper is organized as follows: The second section describes the mathematical model and the notation used in this work. Then, the equations are re-written obtaining an elliptic–hyperbolic decoupling and the details of two discretization strategies are presented. Section four completes the description of the basic discretizations with a discussion on the time integration schemes along with boundary condition treatment and friction. In section five, the dispersion behavior of both the spatial and temporal discretizations is analyzed in detail, while two alternative ways of embedding wave breaking are proposed in section six. Finally, in section seven, the performance of the proposed methodology is extensively validated against experimental measurements from a series of relevant benchmark problems.

2. The physical model

In this work we refer to the improved Green–Naghdi (GN) system of equations in the form proposed in [12]. This formulation has been recovered by adding some terms of $O(\mu^2)$ to the momentum equation in order to improve the frequency dispersion description of the original GN model. In the following we use the notation sketched in Fig. 1, thus we denote by $h(x,t) = h_0 + \eta(x,t) - b(x)$ the total water depth and by $u(x,t)$ the flow velocity (being $\eta(x,t)$ the free surface elevation with respect to the water rest state, h_0 a reference depth and $b(x)$ the topography variation).

The system of equations can be written in its one-dimensional form as:

$$h_t + (hu)_x = 0 \tag{1}$$

$$(I + \alpha\mathcal{T})\left[(hu)_t + (hu^2)_x + g\frac{\alpha-1}{\alpha}h\eta_x\right] + \frac{g}{\alpha}h\eta_x + h\mathcal{Q}(u) = 0$$