Journal of Computational Physics 309 (2016) 18–36

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

A hybrid method with deviational particles for spatial inhomogeneous plasma ☆

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ARTICLE INFO

Article history:
Received 13 October 2015
Received in revised form 9 December 2015
Accepted 23 December 2015
Available online 31 December 2015

Keywords:
Deviational particles
Monte Carlo methods
Plasma physics
Vlasov–Poisson–Landau system
Macro–micro projection
Hybrid methods

ABSTRACT

In this work we propose a Hybrid method with Deviational Particles (HDP) for a plasma modeled by the inhomogeneous Vlasov–Poisson–Landau system. We split the distribution into a Maxwellian part evolved by a grid based fluid solver and a deviation part simulated by numerical particles. These particles, named deviational particles, could be both positive and negative. We combine the Monte Carlo method proposed in [31], a Particle in Cell method and a Macro–Micro decomposition method [3] to design an efficient hybrid method. Furthermore, coarse particles are employed to accelerate the simulation. A particle resampling technique on both deviational particles and coarse particles is also investigated and improved. This method is applicable in all regimes and significantly more efficient compared to a PIC–DSMC method near the fluid regime.

1. Introduction

The evolution of a class of collisional plasmas can be modeled by the spatially inhomogeneous Vlasov–Poisson–Landau (VPL) system, with the electromagnetic fields and applied magnetic fields absent,

$$\partial_t f + \mathbf{v}\cdot\nabla_{\mathbf{x}} f - \mathbf{E}\cdot\nabla_{\mathbf{v}} f = Q_L(f,f), \tag{1a}$$

$$-\nabla_{\mathbf{x}}\cdot\mathbf{E} = \rho(t,\mathbf{x}) = \int f(t,\mathbf{x},\mathbf{v})\,\mathrm{d}\mathbf{v}. \tag{1b}$$

Here $f(t,\mathbf{x},\mathbf{v})$ is the time dependent density distribution function of the charged particles (electrons and ions) in the phase space $(\mathbf{x},\mathbf{v})$, with $\mathbf{x}\in\mathbb{R}^3$ the spatial coordinates and $\mathbf{v}\in\mathbb{R}^3$ the velocity coordinates. The left hand side of (1a) describes the advection of particles under the electric field $\mathbf{E}$, which is solved by the Poisson equation (1b). On the right hand side, the Landau (or Landau–Fokker–Planck) operator,

$$Q_L(f,f) = \frac{A}{4}\frac{\partial}{\partial v_i}\int_{\mathbb{R}^3} u\sigma_{tr}(u)(u^2\delta_{ij}-u_iu_j)\left(\frac{\partial}{\partial v_j}-\frac{\partial}{\partial w_j}\right) f(\mathbf{w})f(\mathbf{v})\,\mathrm{d}\mathbf{w} \tag{2}$$

models the binary collisions due to the long range Coulomb interaction. Here $u=|\mathbf{v}-\mathbf{w}|$, $A=2\pi\left(\frac{e^2}{2\pi\varepsilon_0 m}\right)^2\log\Lambda$, with e the charge of an individual particle, m its mass, ε_0 the permittivity of free space, and $\log\Lambda$ the Coulomb logarithm.

☆ This research was supported by DOE DE-FG02-13ER26152/DE-SC0010613.
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http://dx.doi.org/10.1016/j.jcp.2015.12.050
0021-9991/

The small angle collisions dominate in Coulomb interaction. The Landau operator (2) is derived as the grazing limit of Boltzmann operator (see [10] for example) by Landau in 1936 [20]. We refer the reader to Villani [29] and the references therein for a review of the mathematical derivation. The Landau operator inherits many properties of the Boltzmann operator. First, the density, momentum and energy are conserved during the collision process, i.e.

$$\langle \phi Q \rangle = 0, \quad \text{for } \phi(\mathbf{v}) = 1, \mathbf{v}, |\mathbf{v}|^2/2,$$

where $\langle\cdot\rangle$ is defined as $\langle\psi\rangle = \int_{\mathbb{R}^3} \psi(\mathbf{v})\,d\mathbf{v}$.

Furthermore, the entropy is dissipated

$$\langle Q \log f \rangle \leq 0,$$

with the equal sign holds if and only if f is the local Maxwellian

$$M(t,\mathbf{x},\mathbf{v}) = \frac{\rho(t,\mathbf{x})}{(2\pi T(t,\mathbf{x}))^{3/2}} e^{-\frac{|\mathbf{v}-\mathbf{u}(t,\mathbf{x})|^2}{2T(t,\mathbf{x})}}. \tag{3}$$

Here the density ρ, macroscopic velocity $\mathbf{u}$ and temperature T are determined by f via

$$\langle \phi M(t,\mathbf{x},\mathbf{v}) \rangle = \langle \phi f(t,\mathbf{x},\mathbf{v}) \rangle, \quad \text{for every } \mathbf{x}, t, \tag{4}$$

with $\phi = 1, \mathbf{v}, |\mathbf{v}|^2/2$.

The numerical schemes for the VPL system (1) have been studied extensively. Depending on how to represent the velocity space, these methods can be categorized into two classes, the deterministic (also known as grids based or continuum) methods and probabilistic (also known as particles based) methods. Various techniques have been developed for deterministic methods. We refer the user to [15] and the references therein for a recent review. However all the deterministic methods are suffered from the *curse of dimensionality*. The full system (1) describes a seven dimensional problem (3D in space, 3D in velocity and 1D in time), which leads to tremendous cost of grids based methods.

In contrast, the probabilistic methods, which use particles to represent the velocity space, have the advantage of dimension independence. These methods first studied by Bird [4], are known as Direct Simulation Monte Carlo (DSMC) methods, for rarefied gas. In plasma simulation, Particle in Cell (PIC) methods [13,17,5] are widely used in solving the collisionless Vlasov–Poisson system (i.e. system (1) with $Q_L = 0$ in the right hand side). The Landau operator (2) is solved by Monte Carlo methods, such as Takizuka–Abe method [28] (henceforth TA) and Nanbu's method [22]. Bobylev and Nanbu [6], Bobylev and Potapenko [7] proposed a general framework in designing particle methods for (2). The PIC–DSMC methods, which combine the PIC methods for the advection part and DSMC methods for the collision part, have been widely used in solving the VPL system (1).

However the particle methods become very inefficient near the fluid regime, where the distribution f is close to the local equilibrium M. The reason is that the computation is mainly spent on the collisions between particles sampled from M. These collisions have no net effect since $Q_L(M,M) = 0$. A hybrid method is more favorable in this regime, which decomposes the distribution as

$$f(t,\mathbf{x},\mathbf{v}) = M(t,\mathbf{x},\mathbf{v}) + f_d(t,\mathbf{x},\mathbf{v}). \tag{5}$$

The equilibrium M is evolved by a fluid solver and the deviation f_d is solved by particles. Here we allow $f_d < 0$ and use positive and negative *deviational* particles to represent f_d.

There have been several related works. In gyrokinetic simulations, the so-called δf methods [8,21,16,14] have been proven to be efficient in collisionless regime. Hybrid methods with weighted deviational particles have also been designed when the collisions are modeled by BGK operator [12] and linear Fokker–Planck operator [27]. However all these methods are very difficult to efficiently handle binary collisions. Recently Caflisch et al. [9] and Ricketson et al. [25] studied the thermalization/dethermalization methods with $f_d(\mathbf{v}) \geq 0$ for the spatial homogeneous Coulomb collisions. However they are difficult to be generalized to the inhomogeneous system (1) without allowing f_d to be negative. For rarefied gas, Hadjiconstantinou and co-workers [1,30,2,18,19,23] have studied a low-variance deviational simulation Monte Carlo (LVDSMC) method in which f_d could be negative. However, as discussed in Section 2.1.2, the extension to long range Coulomb collisions leads to dramatic growth of particle numbers and hence a numerical instability.

A different Monte Carlo method with deviational particles for Coulomb collisions has been designed by the authors in [31]. By introducing coarse particles as a rough approximation of f, a large number of collisions are combined efficiently and the growth of particle numbers is reduced from the *numerical* scale to *physical* scale. In this work we seek to extend the method in [31] to the inhomogeneous VPL system (1).

The goal of this work is to design a hybrid method with deviational particles which (a) is applicable in all regimes and (b) is much more efficient than the PIC–DSMC method near the fluid regime. The starting point is an operator splitting between the collision part and the advection part. The collision part is solved as in the homogeneous case cell by cell. As for the advection part, we combine a macro–micro projection method [3] and a Particle-In-Cell method to evolve the macroscopic quantities and all particles. As a result, the number of particles grows in the physical scale and a particle resampling method is applied on both deviational and coarse particles to further control the growth.