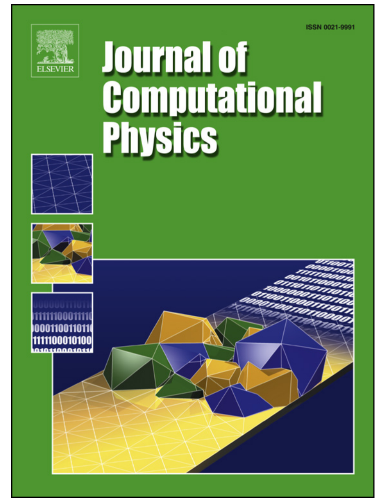
Accepted Manuscript



A robust HLLC-type Riemann solver for strong shock

Zhijun Shen, Wei Yan, Guangwei Yuan

PII: S0021-9991(16)00002-4
DOI: http://dx.doi.org/10.1016/j.jcp.2016.01.001
Reference: YJCPH 6340

To appear in: *Journal of Computational Physics*

Received date: 7 July 2015
Revised date: 28 December 2015
Accepted date: 1 January 2016

A Robust HLLC-type Riemann Solver for Strong Shock

Zhijun Shen[1,2*], Wei Yan[1], Guangwei Yuan[1]

1. National Key Laboratory of Science and Technology on Computational Physics, Institute of Applied Physics and Computational Mathematics, P. O. Box 8009-26, Beijing 100088, China
2. Center for Applied Physics and Technology, HEDPS, Peking University

Abstract

It is well known that for the Eulerian equations the numerical schemes that can accurately capture contact discontinuity usually suffer from some disastrous carbuncle phenomenon, while some more dissipative schemes, such as the HLL scheme, are free from this kind of shock instability. Hybrid schemes to combine a dissipative flux with a less dissipative flux can cure the shock instability, but also may lead to other problems, such as certain arbitrariness of choosing switching parameters or contact interface becoming smeared. In order to overcome these drawbacks, this paper proposes a simple and robust HLLC-type Riemann solver for inviscid, compressible gas flows, which is capable of preserving sharp contact surface and is free from instability. The main work is to construct a HLL-type Riemann solver and a HLLC-type Riemann solver by modifying the shear viscosity of the original HLL and HLLC methods. Both of the two new schemes are positively conservative under some typical wavespeed estimations. Moreover, a linear matrix stability analysis for the proposed schemes is accomplished, which illustrates the HLLC-type solver with shear viscosity is stable whereas the HLL-type solver with vorticity wave is unstable. Our arguments and numerical experiments demonstrate that the inadequate dissipation associated to the shear wave may be an unique reason to cause the instability.

Keywords: Godunov methods, HLL scheme, HLLC scheme, Numerical shock instability, Stability analysis

1. Introduction

The approximate Riemann solvers are popular shock capturing methods and are widely used in computational studies for high speed flows. However, using these schemes in multidimensional calculation may suffer from a numerical shock instability problem. The shock instability phenomenon was first reported in 1988 by Peery and Imlay [24] as they computed the supersonic flow field around a blunt body using Roe's scheme. Since the instability mechanism may ruin all efforts to compute grid-aligned shock waves, from then on, many researchers began to concern this problem and several attempts have been made to understand and cure the phenomenon.

Of all modification methods, the most popular one to prevent such numerical deficiency is a hybrid method proposed by Quirk [26]. He noticed that some schemes with contact resolving exactly always show carbuncle phenomena while other more dissipative ones are free from such unstable failings. A typical example is that the less dissipative HLLC scheme [35] is unstable whereas the more dissipative formulation HLLE [10] exhibits strong robustness. He suggested to combine these methods by using a dissipative scheme in shock region and a less dissipative scheme elsewhere. From then, similar ideas have been used extensively although the concrete combination ways could be remarkably different [21] [11] [37] [14]. Such combination really avoid the carbuncle phenomenon, however, it brings other problems simultaneously. For example, the strict Rankine-Hugoniot jump conditions of many hybrid fluxes might be broken across a discontinuity due to the hybridization of complementary solvers [21] [11] [37]. In particular, the contact interface might be smeared. In addition, the choice of switching parameters required in the hybrid schemes

*Corresponding author.Tel.: 86-10-61935178.
Email addresses: shen_zhijun@iapcm.ac.cn (Zhijun Shen[1,2]), wyanmath01@sina.com (Wei Yan[1]), yuan_ guangwei@iapcm.ac.cn (Guangwei Yuan[1])