Journal of Computational Physics 308 (2016) 411–420

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

Efficient formulation of scale separation for multi-scale modeling of interfacial flows

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ARTICLE INFO

Article history:
Received 24 March 2015
Received in revised form 12 November 2015
Accepted 20 November 2015
Available online 18 December 2015

Keywords:
Multi-scale
Multi-phase
Interfaces
Scale separation
Level-set method

ABSTRACT

We propose an efficient formulation of the scale-separation approach which has been developed by Han et al. [10] for multi-scale sharp interface modeling of multi-phase flows based on the level-set technique. Instead of shifting the entire level-set field twice as in the original method, the improved method identifies the non-resolved interface structures from two auxiliary level-sets close to the interface. Non-resolved structures are separated from the interface by a localized re-distancing method, which increases the computational efficiency considerably compared to the original global reinitialization procedure. Several tests for two-phase flow problems, involving simple and complex interface structures, are carried out to show that the present method maintains sharper interface structures than the original method, and achieves effective scale-separation.

1. Introduction

Simulations of two-phase flow, such as bubble interaction [17], drop impact [7] and spray atomization [2,15], need to resolve length scales that can span several orders of magnitude, which poses a great computational challenge. Adaptive mesh refinement (AMR) [23] and multi-resolution (MR) methods [11,9], even with local time stepping, do not sufficiently reduce computational cost to enable accurate routine simulations of complex interfacial flows. A promising approach to improve efficiency is offered by multi-scale sharp-interface methods [18,14,10]. One essential procedure in multi-scale modeling is scale separation. For a given spatial resolution with grid size h, the interface segments with characteristic size δ can be categorized as resolved if $\delta > h$, or non-resolved if $\delta < h$. In previous work, two approaches have been proposed for scale separation: one is based on the refined level-set grid method (RLSG) [13,18,14], the other is the constrained stimulus–response procedure (CSRP) [10]. While RLSG requires a two-grid system where a higher-resolution grid is used for representing the interface, CSRP uses a single grid for representing both the interface and the individual fluids. CSRP identifies the resolved and non-resolved interface segments based on the different responses they exhibit when subjected to a small shift of the level-set field, and separates these scales with a two-step level-set re-initialization procedure. Although CSRP is effective, in particular the additional re-initialization operation significantly increases computational cost.

In this paper, an improved scale-separation method is proposed to increase computational efficiency. Using two auxiliary level-set fields instead of level-set shifting, the new method identifies non-resolved interface structures by examining the topological consistency between the auxiliary and zero level-sets. Non-resolved structures are subsequently separated by

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http://dx.doi.org/10.1016/j.jcp.2015.11.044
0021-9991/

a localized re-distancing approach to avoid additional re-initialization operations. The performance of the new method is evaluated through several tests, including single-vortex flow, underwater explosion, shock–bubble interaction, and liquid-filament breakup.

2. Sharp-interface method

The governing flow equations can be written as

$$\frac{\partial \mathbf{U}}{\partial t} + \nabla \cdot \mathbf{F} = \nabla \cdot \mathbf{F}_v, \tag{1}$$

where $\mathbf{U}$ represents the density of mass, momentum and total energy; $\mathbf{F}$ the convective fluxes, and $\mathbf{F}_v$ the viscous fluxes.

To close the governing equations, an equation of states (EOS) must be defined. For an ideal gas, the pressure is determined from

$$p = (\gamma - 1)\rho e, \tag{2}$$

where γ is the ratio of specific heats, ρ is mass density and e internal energy density. For water-like fluids, we use Tait's equation to provide a relation,

$$p = B[(\frac{\rho}{\rho_0})^\gamma - 1] + p_0, \tag{3}$$

where ρ_0 and p_0 are the density and pressure of the liquid at the reference condition, respectively, and B is a parameter. For two-phase flows the above governing equations are solved by a conservative-sharp interface method [17,22], where the underlying conservative scheme on a Cartesian grid is modified by considering computational cells cut by the interface. These methods are not only able to cope with violent compressible flows involving strong interface interactions, but also can handle incompressible flow with large density and viscosity ratios and surface tension effects.

For tracking the phase-interface we employ the level-set function, φ, which describes the signed distance from the interface to each cell center [6]. The zero-level-set, $\varphi = 0$, represents the interface Γ. The entire domain is divided into two sub-domains, for two fluids indicated by different signs of the level-set function. The level-set field is propagated by

$$\frac{\partial \varphi}{\partial t} + \mathbf{u} \cdot \nabla \varphi = 0, \tag{4}$$

where $\mathbf{u}$ represents the evolution velocity of the level-set [17]. In practice, the level-set is updated only in a narrow band near the interface, which usually includes nearest cell-layers within $|\varphi| < 4h$. The entire level-set field is re-initialized at every time-step by the following equation [6]

$$\frac{\partial \varphi}{\partial \tau} + \mathrm{sgn}\,(\varphi)\,(|\nabla \varphi| - 1) = 0, \tag{5}$$

in order to maintain the signed-distance property of the level-set, where τ is a fictitious time. To update the flow field, the governing equations of each fluid are discretized by a 5th-order WENO-LLF scheme [19] and a 2nd-order TVD Runge–Kutta time integration [27]. Advection and reinitialization of the level-set field are discretized, respectively, by a 5th-order WENO upwind scheme and a 1st-order upwind scheme [6]. The time-step is set according to Ref. [31],

$$\Delta t = CFL \cdot \min\left(\frac{h}{|u| + c}, \frac{\rho h^2}{\mu}, \sqrt{\frac{\rho}{8\pi\sigma}h^{\frac{3}{2}}}\right), \tag{6}$$

where c is the sound speed. We set CFL = 0.6 for all our simulations. In addition, a wavelet-based adaptive multi-resolution algorithm is adopted for improving computational efficiency [9]. Due to the application of a storage-and-operation-splitting pyramid data structure, this method is able to achieve high memory and computational efficiency. All computations have been conducted on a workstation with 4 quad-core Intel Xeon Processor $E5620$ processors (12M Cache, 2.4 GHz) with 24 GB of RAM. The Intel Threading Building Blocks (TBB) library [4] is used to map logical tasks to physical threads.

3. Scale separation

Non-resolved interface segments are generated during the interface evolution, and are prone to introduce numerical fluctuations or even numerical instabilities, especially for high-resolution simulations of inviscid compressible multi-phase flow problems using the sharp-interface model [10]. They can also lead to unphysical results due to scale-dependent dynamics [18]. An effective approach is to separate them from the resolved scales and treat them differently. For example, in Ref. [10], the well-resolved interface is handled by a sharp interface method, non-resolved interface segments are converted into Lagrangian particles, or they are merged into the opposite phase to maintain mass conservation, or they are deleted.