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Assessment of a hybrid finite element and finite volume code for turbulent incompressible flows ☆

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ABSTRACT

Hydra-TH is a hybrid finite-element/finite-volume incompressible/low-Mach flow simulation code based on the Hydra multiphysics toolkit being developed and used for thermal-hydraulics applications. In the present work, a suite of verification and validation (V&V) test problems for Hydra-TH was defined to meet the design requirements of the Consortium for Advanced Simulation of Light Water Reactors (CASL). The intent for this test problem suite is to provide baseline comparison data that demonstrates the performance of the Hydra-TH solution methods. The simulation problems vary in complexity from laminar to turbulent flows. A set of RANS and LES turbulence models were used in the simulation of four classical test problems. Numerical results obtained by Hydra-TH agreed well with either the available analytical solution or experimental data, indicating the verified and validated implementation of these turbulence models in Hydra-TH. Where possible, some form of solution verification has been attempted to identify sensitivities in the solution methods, and suggest best practices when using the Hydra-TH code.

1. Introduction

Hydra-TH [1–9] refers to the hybrid finite-element/finite-volume incompressible/low-Mach flow solver in the Hydra toolkit being used for thermal-hydraulics ("TH") applications for the Consortium for Advanced Simulation of Light Water Reactors ("CASL") [10]. The Hydra-TH development, verification, and validation is supported under the U.S. Department of Energy Contract No. DE AC05-00OR22725. Hydra-TH features a recently developed second-order, hybrid finite-element/finite-volume, incremental projection algorithm for time-dependent incompressible viscous flows [11]. The algorithm circumvents the usual div-stability constraints, i.e., does not require explicit treatment of troublesome pressure modes using Rhie–Chow interpolation or a pressure-stabilized Petrov–Galerkin formulation. The use of a co-velocity and high-resolution advection scheme with consistent edge-based treatment of viscous/diffusive terms yields a robust algorithm for a broad spectrum of incompressible flows. A detailed description of the numerical algorithms, implementations, and basic verification and validation problems has been presented in the work of Christon et al. [11].

Hydra-TH was built as one of a number of applications using the Hydra multiphysics toolkit. The Hydra toolkit was written in C++, and it provides a rich suite of components that permits rapid application development, supports multiple

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1 This work was conducted when the lead author was at North Carolina State University.

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discretization techniques, and provides I/O interfaces to permit reading/writing multiple file formats for meshes, plot data, time-history, surface-based and restart output. Data registration is used to provide the ability to register variables at appropriate locations (e.g., node, element, dual-edge, etc.), and provides integrated and automatic output and restart capabilities along with memory management. The toolkit also provides run-time parallel domain decomposition with data-migration for both static and dynamic load balancing. Linear algebra is handled through an abstract virtual interface, which makes it possible to use some of the most popular libraries such as PETSc [12] and Trilinos [13]. The use of output delegates provides the ability to develop lightweight physics-specific output kernels with minimal memory overhead that can be tailored to a specific physics, e.g., computation of vorticity, helicity, enstrophy for large-eddy simulations. Verification testing is part of the Hydra software control process and ensures that Hydra-TH is solving problems of interests to the CASL project and meeting design requirements. It is one component of a larger testing infrastructure. This paper is aimed at identifying verification problems and the Hydra-TH solutions to those problems. More test problems will be added to the suite of verification and validation for flow simulation, as the Hydra-TH code will change over time.

The rest of this paper is organized as follows: Section 2 first introduces the baseline numerical method employed by Hydra-TH, and then the Reynolds-averaged Navier–Stokes ("RANS") and large-eddy simulation ("LES") turbulence models implemented in the current suite. Section 3 presents four classical test cases computed by Hydra-TH: 1) the steady-state Poiseuille flow; 2) a laminar flow past a flat plate at Reynolds number of $Re = 100{,}000$, which can be compared with the Blasius solution; 3) a wall-bounded turbulent channel flow at $Re_\tau = 590$, where two RANS turbulence models are verified and validated according to the referenced Direct Numerical Simulation (DNS) data; 4) large-eddy simulation of a lid-driven cavity flow, in which three LES models in Hydra-TH are tested and verified with respect to the referenced experimental data. Section 4 gives a conclusion of the work in this paper, and outlook for future work.

2. Numerical method

Hydra-TH uses a hybrid finite-element/finite-volume discretization for the incompressible/low-Mach Navier–Stokes equations. All the transport variables are cell-centered and treated with a conservative discretization that includes a high-resolution monotonicity-preserving advection algorithm. In Hydra-TH, the spatial discretization is formally derived using a discontinuous Galerkin ("DG") framework that, in the limit, reduces to a locally conservative finite volume method [11] (also see [14] as an example of high-order DG implementation). In order to provide compatibility with modern piecewise linear interface construction (PLIC) based volume-tracking algorithms that are based on a finite volume formalism, we choose piecewise-constant test functions for the compressible and incompressible flow solvers in the Hydra toolkit. However, it should be noted that this choice does not restrict the current methods or software from extension to include higher-order (higher than linear) reconstruction techniques or the incorporation of higher-order test functions. In addition, this choice simplifies sharing a common discretization for both the compressible and incompressible flow solvers.

In the present work, the high-resolution advection algorithm is designed to permit both implicit and explicit advection with the explicit advection targeted primarily at volume tracking with interface reconstruction. The time-integration methods include backward-Euler and neutrally dissipative trapezoidal method. The implicit advective treatment delivers unconditional stability for the linear scalar transport equations, and conditional stability for the momentum transport equations. A sharp stability estimate for the momentum equations is not available, but operational experience shows that the upper stability range is on the order of CFL = 20 to CFL = 40 for steady-state problems, and of CFL = 5 to CFL = 10 for time-accurate problems. Note that for the steady-state problems, the backward-Euler time integration provides an additional damping that, in conjunction with the upper stability range of CFL numbers, provides a computationally efficient solution method. For the Unsteady Reynolds–Averaged Navier–Stokes ("URANS") and Large-Eddy Simulation ("LES") computations, the trapezoid rule is neutrally dissipative, and delivers optimal performance for the more stringent CFL requirements for time-accurate flow.

Hydra-TH is designed to incorporate both the well established and state-of-the-art turbulence models, ranging from the traditional RANS, through the LES, to the hybrid RANS/LES models. The following models implemented in Hydra-TH have been used in this work:

- RANS models
  a. The Spalart–Allmaras ("S–A") one-equation eddy viscosity model developed by Spalart and Allmaras [15]. It is important to include this model besides its reliability in complex flows [16], since the most popular hybrid RANS/LES model (DES) is built around this model. It is important to mention that DES has gained significant attention due to its potential for predicting complex unsteady turbulent flows at low computational cost [17].
  b. The Re-Normalized Group ("RNG") k–ε two-equation model developed by Yakhot et al. [18], along with the use of a wall function. For the current implementation, the "law-of-the-wall" approach has been used to provide near-wall modeling avoiding the need for highly resolved boundary layer meshes. Due to its limitations in usage, e.g. recirculating flows, the standard law-of-the-wall was modified based on a new scale for the friction velocity following the method proposed by Launder and Spalding [19]. The modified law-of-the-wall reduces to the standard law-of-the-wall under the conditions of uniform wall shear stress and when the generation and dissipation of turbulent kinetic energy are in balance (i.e. when the turbulence structure is in equilibrium). Under such conditions, $u^* \approx u^+$ and thus, $y^* \approx y^+$.