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A spectrally accurate method for overlapping grid solution of incompressible Navier–Stokes equations

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ABSTRACT

An overlapping mesh methodology that is spectrally accurate in space and up to third-order accurate in time is developed for solution of unsteady incompressible flow equations in three-dimensional domains. The ability to decompose a global domain into separate, but overlapping, subdomains eases mesh generation procedures and increases flexibility of modeling flows with complex geometries. The methodology employs implicit spectral element discretization of equations in each subdomain and explicit treatment of subdomain interfaces with spectrally-accurate spatial interpolation and high-order accurate temporal extrapolation, and requires few, if any, iterations, yet maintains the global accuracy and stability of the underlying flow solver. The overlapping mesh methodology is thoroughly validated using two-dimensional and three-dimensional benchmark problems in laminar and turbulent flows. The spatial and temporal convergence is documented and is in agreement with the nominal order of accuracy of the solver. The influence of long integration times, as well as inflow–outflow global boundary conditions on the performance of the overlapping grid solver is assessed. In a turbulent benchmark of fully-developed turbulent pipe flow, the turbulent statistics with the overlapping grids is validated against published available experimental and other computation data. Scaling tests are presented that show near linear strong scaling, even for moderately large processor counts.

1. Introduction

Finding numerical solutions to partial differential equations (PDEs) by decomposing the computational domain into smaller subdomains is an idea that has been around for well over a century. Domain decomposition methods have been utilized for several different purposes, including straightforward parallelization [1–4], simplified mesh generation for complex geometries [5–8], and the ability to use different parameters or methods in different subdomains [9–12]. These techniques exist in many forms, and each has its strengths. Some decompose the global domain into overlapping subdomains [13–18], while others employ non-overlapping subdomains [19,9,7,20–22]. Some use explicit interpolation techniques for values at interface boundaries [9,23,20,24], and others carry out implicit interpolation [25,26,22,27,13,14,28–30]. Domain decomposi-

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tion techniques have been developed for use with several numerical methods including finite difference [14,5,9,23], finite element [25], finite volume [17,31], and spectral methods [21,22].

The earliest known research in domain decomposition methods was performed by H.A. Schwarz whose work was published in 1870 [13]. The original Schwarz Alternating Method, initially proposed for analytical calculations [32], was developed for the global solution of boundary value problems for harmonic functions [15] decomposed into overlapping subdomains, $\Omega = \Omega_1 \cup \Omega_2$. The solution in the first subdomain (Ω_1 with boundaries $\partial\Omega_1 \cap \partial\Omega$ and $\Gamma_1 = \partial\Omega_1 \setminus \partial\Omega$) is found using the global boundary conditions on $\partial\Omega_1 \cap \partial\Omega$ and corresponding values from Ω_2 at the previous iteration on Γ_1. The solution of Ω_2 is then found by using values from the solution in Ω_1 on Γ_2. These two steps are iterated until sufficient convergence is reached (see [33,32,34]).

In the 1960's, Volkov generalized the original Schwarz Alternating Method into a numerical domain decomposition technique, in a form of the *Composite Mesh Difference Method* (CMDM) [14]. CMDM used finite difference methods to solve the 2-dimensional Poisson equation numerically on overlapping grids. His research laid the foundation for subsequent techniques that extended the use of CMDM to other elliptical and hyperbolic PDEs, and boundary value and initial value problems, with the ability to use curvilinear meshes (see [35–38,6]). Overlapping domain decomposition methods have also been developed to model complex equations and handle various difficulties in solving practical problems. The Chimera Grid Scheme, introduced in [39], employs overset (overlapping) grids for modeling flows in complex geometries. Shortly after initial development it was enhanced for use with three dimensional flows modeled by the Euler equations [40] and later with the addition of the thin-layer Navier–Stokes equations. More recently, Chimera Grid techniques have been used to model a variety of problems with complex geometries [41,42]. Subsequently, Henshaw and Schwendeman [17,4] developed a method for using overlapping mesh techniques in modeling high-speed reactive flows, in two and three dimensions.

In addition, techniques that employ non-overlapping grids (sometimes called patched grids) were developed. Examples include a zonal approach that uses a flux-vector splitting technique for the determination of interface values in Euler equations [43–46], Lions method [19] that uses an iterative technique to arrive at the correct values to be passed between non-overlapping subdomains in solving Laplace's equation and more general second-order elliptic problems, Dawson's approach [9] that solves the heat equation using an explicit finite difference formula to determine the interface values and allows for different time stepping to be used in different subdomains. Non-overlapping grid techniques have also been extended and employed in solving the advection–diffusion equation [20] and the Navier–Stokes equations [45,22]. Some of the more recently developed non-overlapping domain decomposition methods achieve high finite global accuracy [47] and some spectral accuracy [21,7,48,22].

While non-overlapping mesh methods allow some flexibility in mesh generation, the constraints in these techniques inhibit additional flexibility that is seen in overlapping mesh methods. By allowing variable overlap size, a broad range of potential mesh configurations are supported with overlapping methods, thus allowing for more simplified mesh generation. Additionally, overlapping methods provide a convenient framework for further extension towards moving domain methods, allowing for general and unconstrained motion of rigid body parts through the background stationary meshes [49–52].

So far, existing overlapping grid methods for the time-dependent PDE coupling have been traditionally relying on low-order, finite-difference or finite-volume schemes. Although some of the methods have been extended to achieve higher-order spatial convergence, using extended stencil finite-difference or compact schemes, the upper bound of the global accuracy has been usually limited to four [23,24,53], and at most six [54–56,27,57]. Recently, Brazell, Sitaraman and Mavriplis developed a high-order overlapping Discontinuous Galerkin solver for compressible equations, that uses Lagrangian interpolation at interface boundaries, and documented a polynomial convergence up to fourth order [58]. In the current paper, we introduce a spectrally-accurate overlapping mesh method for incompressible equations, that is based on a spectral-element method. The Spectral Element Method, which can be perceived as a high-order extension of the Finite Element Method, divides a domain into several conforming and adjacent elements [59,3,60]. The volume within each element is discretized using Nth-order tensor-product Lagrange interpolating polynomials on Gauss–Lobatto–Legendre nodal points. Approximations in all elements are coupled at the boundaries to form a global solution [60], which achieves spectral convergence with p- (polynomial order) refinement. In this work, we combine a Spectral Element Method solver with the overlapping grid approach, to arrive at a globally spectrally-accurate method for solution of the incompressible Navier–Stokes equations on overlapping domains.

One of the inherent challenges with overlapping grid methods is to minimize the errors that are introduced due to the coupling of the individual subdomain solutions into the global solution. The coupling errors consist of spatial errors and temporal errors, and have to be treated separately. Spatial errors are introduced by the spatial interpolation stencil employed to obtain a function value at the interface points of one domain from the gridpoint values in the adjacent domains at the same time level. Some overlapping mesh methods circumvent the spatial error by requiring that the gridpoints in overlapping domains exactly coincide [61,62], thus fully conserving communicated information, with the drawback of decreased flexibility in mesh generation. Other methods that do not require the exact match of the gridpoints and thus are more flexible, use finite order interpolation schemes to determine values from adjacent subdomains. Although simple linear interpolation techniques have been popular [6,9,63,29,64], it was shown by Chesshire and Henshaw [6] that an interpolation scheme should be consistent with the accuracy of the underlying solver and higher-order interpolation is required to maintain the accuracy of the coupled solution with high-order methods, generally, of the order of one higher than the underlying solver accuracy if the overlap width scales with the grid resolution, and the same if it stays constant during grid refinement. Thus, in fourth- and sixth-order methods [58,23,27,24], a generalized Lagrangian interpolation method