Journal of Computational Physics 307 (2016) 460–475

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

Second-order accurate finite volume method for well-driven flows

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ARTICLE INFO

Article history:
Received 10 December 2013
Received in revised form 19 November 2015
Accepted 9 December 2015
Available online 15 December 2015

Keywords:
Finite volume method
Near-well modeling
Groundwater
Flow simulations
Second-order accuracy

ABSTRACT

We consider a finite volume method for a well-driven fluid flow in a porous medium. Due to the singularity of the well, modeling in the near-well region with standard numerical schemes results in a completely wrong total well flux and an inaccurate hydraulic head. Local grid refinement can help, but it comes at computational cost. In this article we propose two methods to address the well singularity. In the first method the flux through well faces is corrected using a logarithmic function, in a way related to the Peaceman model. Coupling this correction with a non-linear second-order accurate two-point scheme gives a greatly improved total well flux, but the resulting scheme is still inconsistent. In the second method fluxes in the near-well region are corrected by representing the hydraulic head as a sum of a logarithmic and a linear function. This scheme is second-order accurate.

1. Introduction

The stationary groundwater flow equation is obtained by substituting the Darcy law

$$\mathbf{u} = -\mathbb{K}\nabla h \quad \text{in} \quad \Omega \tag{1}$$

into the continuity equation

$$\nabla \cdot \mathbf{u} = g_s, \tag{2}$$

where $\mathbf{u}$ is the Darcy velocity, g_s describes sources and sinks, $\mathbb{K}$ is the hydraulic conductivity tensor, h is the hydraulic head, and $\Omega \subset \mathbb{R}^3$ is a bounded domain. In this paper we assume that the hydraulic conductivity is isotropic, so that $\mathbb{K} = KI$.

We consider the following boundary conditions:

$$h = g_D \quad \text{on} \quad \Gamma_D, \tag{3}$$

$$\mathbf{u} \cdot \mathbf{n} = g_N \quad \text{on} \quad \Gamma_N, \tag{4}$$

where $\partial\Omega = \Gamma_D \cup \Gamma_N$ is the domain boundary, $\Gamma_D \cap \Gamma_N = \emptyset$, $\Gamma_D \neq \emptyset$, $\Gamma_D = \bar{\Gamma}_D$ and $\mathbf{n}$ is a unit vector normal to $\partial\Omega$ pointing outwards.

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http://dx.doi.org/10.1016/j.jcp.2015.12.021
0021-9991/

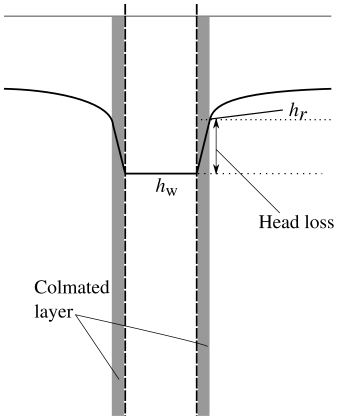

Fig. 1. Head loss due to colmated layer.

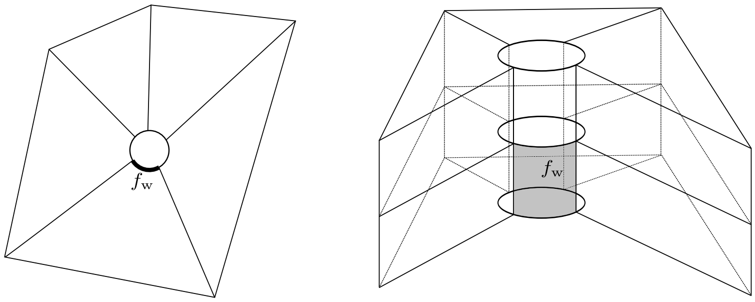

Fig. 2. Well in two (left) and three (right) dimensions.

A colmated layer, also known as the skin effect, is formed along well walls due to well clogging [1,2]. This causes an additional hydraulic resistance (see Fig. 1). As a result, the flux density through the well filter is

$$u = \Psi(h_r - h_w), \tag{5}$$

where: h_w is the hydraulic head inside the well, h_r is the hydraulic head just outside the colmated layer (see Fig. 1), r is the well radius, and $\Psi = K_c/d_c$ is the transfer coefficient, while K_c and d_c are the unknown conductivity and thickness of the colmated layer, respectively. The physical colmated layer thickness is assumed to be small, so that this layer can be modeled as an infinitely thin film of finite Ψ.

Hydraulic head varies logarithmically and its gradient changes sharply in the well vicinity (Fig. 1). Thus, linear approximation of hydraulic head is inappropriate on coarse grids and numerical methods based on it are inaccurate in the near-well region.

Accurate modeling in the near-well region is important in reservoir engineering. Flow in the entire reservoir is induced mainly by wells, therefore poor near-well modeling results in accuracy loss throughout the model.

Numerous families of second-order accurate numerical methods are applicable to porous media flows. Here we consider non-linear two-point approximations [3–10]. Although there is no proof that these methods are second-order accurate [11], numerical tests show second-order accuracy for the hydraulic head and first-order accuracy for the fluxes. These schemes preserve positivity of the solution, but at the price of having to solve a non-linear system even when the problem is linear. Nevertheless, linear approximation is deployed and therefore the accuracy is lost on coarse grids if a well is present.

Local grid refinement can alleviate the problem [12]. However, this comes at a computational cost.

Methods for well modeling have been widely discussed in the literature [12–18]. A commonly used method is the Peaceman model [14,17,18]. This approach was originally formulated for finite differences, with a well placed in a cell center. It has been extended to various other discretization methods [13]. Peaceman model introduces an additional equation which yields a greatly improved flow rate, but it does not improve the accuracy of the hydraulic head around the well.

In commonly available mesh generators it is possible to specify points that are guaranteed to become mesh nodes once the mesh is generated. Thus we can easily represent a well as a mesh node in two-dimensional models or as an array of mesh edges in three-dimensional models. For a finite volume code, it is more appropriate to associate a well with a cell in two dimensions or with an array of cells in three dimensions. Therefore, we construct cylinders (circles in two dimensions) around well edges (nodes) as in Fig. 2. Another way to represent a well is described in Example 5.

The *well face correction* method (WFC) described in Subsection 2.1 is related to the Peaceman method and results in a greatly improved well extraction rate compared to the uncorrected scheme, but the hydraulic head is still inconsistent