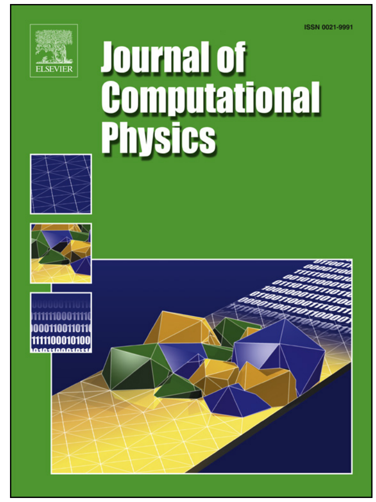

From stochastic processes to numerical methods: A new scheme for solving reaction subdiffusion fractional partial differential equations

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Abstract

We have introduced a new explicit numerical method, based on a discrete stochastic process, for solving a class of fractional partial differential equations that model reaction subdiffusion. The scheme is derived from the master equations for the evolution of the probability density of a sum of discrete time random walks. We show that the diffusion limit of the master equations recovers the fractional partial differential equation of interest. This limiting procedure guarantees the consistency of the numerical scheme. The positivity of the solution and stability results are simply obtained, provided that the underlying process is well posed. We also show that the method can be applied to standard reaction-diffusion equations. This work highlights the broader applicability of using discrete stochastic processes to provide numerical schemes for partial differential equations, including fractional partial differential equations.

1. Introduction

Reaction subdiffusion fractional partial differential equations have been widely used in recent years as mathematical models of systems of particles subject to, trapping, obstacles and reactions [1, 2, 3, 4, 5, 6, 7, 8]. Subdiffusion, characterised by a mean squared displacement of diffusing particles that grows slower

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