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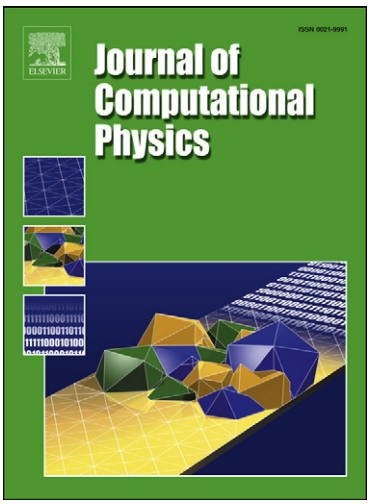

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Anderson acceleration of the Jacobi iterative method: an efficient alternative to Krylov methods for large, sparse linear systems

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Abstract

We employ Anderson extrapolation to accelerate the classical Jacobi iterative method for large, sparse linear systems. Specifically, we utilize extrapolation at periodic intervals within the Jacobi iteration to develop the Alternating Anderson-Jacobi (AAJ) method. We verify the accuracy and efficacy of AAJ in a range of test cases, including nonsymmetric systems of equations. We demonstrate that AAJ possesses a favorable scaling with system size that is accompanied by a small prefactor, even in the absence of a preconditioner. In particular, we show that AAJ is able to accelerate the classical Jacobi iteration by over four orders of magnitude, with speed-ups that increase as the system gets larger. Moreover, we find that AAJ significantly outperforms the Generalized Minimal Residual (GMRES) method in the range of problems considered here, with the relative performance again improving with size of the system. Overall, the proposed method represents a simple yet efficient technique that is particularly attractive for large-scale parallel solutions of linear systems of equations.

Key words: Linear systems of equations, Fixed-point iteration, Jacobi method, Anderson extrapolation, Nonsymmetric matrix, Poisson equation, Helmholtz equation, Parallel computing

1. Introduction

In nearly all areas of computational physics, it is common to encounter linear systems of equations of the form

$$\begin{aligned} &\mathbf{A}\mathbf{x} = \mathbf{b}\,, \\ &\mathbf{A} \in \mathbb{C}^{N\times N}\,,\ \mathbf{x} \in \mathbb{C}^{N\times 1} \text{ and } \mathbf{b} \in \mathbb{C}^{N\times 1}\,, \end{aligned} \tag{1}$$

where $\mathbb{C}$ is the set of all complex numbers. For small systems, solution strategies based on direct methods are typically the preferred choice. However, as the size of the system increases, it becomes necessary to employ iterative approaches in order to efficiently determine the solution. The basic fixed-point techniques that have been developed for this purpose include the Richardson, Jacobi, Gauss-Seidel, and Successive over-relaxation (SOR) methods [1]. However, these approaches suffer from relatively large prefactors and poor scaling with system size. This makes them unattractive for solving large systems of equations compared to Krylov subspace approaches such as the conjugate gradient [2] and Generalized Minimal Residual (GMRES) [3] methods.

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