Journal of Computational Physics 306 (2016) 73–91

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

A hybrid scheme for compressible magnetohydrodynamic turbulence

Yan Yang [a], Minping Wan [b], Yipeng Shi [a,*], Kun Yang [a,*], Shiyi Chen [a,c,d]

[a] State Key Laboratory for Turbulence and Complex Systems, Center for Applied Physics and Technology, College of Engineering, Peking University, Beijing 100871, PR China
[b] Department of Physics and Astronomy, University of Delaware, Newark, DE 19716, USA
[c] Collaborative Innovation Center of Advanced Aero-Engine, Beijing 100191, PR China
[d] Department of Mechanics and Aerospace Engineering, Shenzhen, Guangdong 518055, PR China

ARTICLE INFO

Article history:
Received 24 August 2015
Received in revised form 9 November 2015
Accepted 15 November 2015
Available online 19 November 2015

Keywords:
Magnetohydrodynamics
Hybrid compact-WENO scheme
Compressible turbulence

ABSTRACT

An efficient, high-resolution and oscillation-free hybrid scheme for shock-turbulence interactions in compressible magnetohydrodynamic (MHD) problems is presented. The hybrid scheme couples a sixth-order compact finite difference scheme in the smooth regions with a fifth-order WENO scheme in the shock regions. An eighth-order pentadiagonal filter is derived and utilized to maintain numerical stability and eliminate spurious oscillations. Various numerical examples are presented and the hybrid algorithm is proved to be accurate for smooth solutions and be able to capture discontinuities robustly. We have also shown the good capability of the hybrid scheme for the compressible MHD turbulence.

1. Introduction

Magnetohydrodynamic (MHD) turbulence is a widespread state of plasma in both astrophysical processes like the solar wind and interstellar medium [1,2], and engineering problems such as liquid-metal cooling and plasma confinement [3,4]. The nonlinear interaction among the magnetic field and flow velocity introduces much more degrees of freedom into the MHD system than fluid turbulence, and the existence of shock waves makes the compressible MHD turbulence an even more complicated dynamic system.

The approach of numerical simulation plays an important role in understanding the physical details of compressible MHD turbulence. A large number of papers have been devoted to develop an accurate, robust, and efficient algorithm for the simulation of compressible MHD problems, including the upwind scheme [5–7], flux corrected transport scheme [8], piecewise parabolic method [9], discontinuous Galerkin method [10], and convective upwind and split pressure schemes [11]. However, only a few studies have employed numerical simulation in the study of compressible MHD turbulence [12,13], and the second or third essentially non-oscillatory (ENO) [14] and weighted essentially non-oscillatory (WENO) [15] schemes are adopted for most of these works. The development of a robust and high-resolution numerical method focusing on the compressible MHD turbulence simulation, especially the interaction of turbulence and strong shock waves, remains to be an open issue.

The vast range of spatial and temporal scales in turbulent fields lead to a strict requirement on the small-scale resolution of numerical methods. The high-order compact finite difference (FD) scheme proposed by Lele [16] has a spectral-like

* Corresponding author.
E-mail addresses: syp@mech.pku.edu.cn (Y. Shi), yangkun@pku.edu.cn (K. Yang).

http://dx.doi.org/10.1016/j.jcp.2015.11.025

resolution, and has gained wide popularity in the computation of turbulent flows with no or weak compressible effects [17–21]. However, the compact FD scheme is not suitable for strong shock waves in highly compressed flow regions due to the Gibbs oscillations [22]. To capture the strong discontinuities, one way is to add numerical dissipation at the shock regions, such as the combination of the compact FD scheme with filters [23–28] or hyperviscosity [29–31], which can eliminate the spurious oscillations at the high wavenumber range without affecting resolved waves at lower wavenumbers. Another way is to couple the compact FD scheme in the smooth regions with the WENO scheme [15,32] in the shock regions (the hybrid approach). The WENO scheme is highly dissipative at discontinuities and has shown its effectiveness for robust shock capturing in both compressible fluid dynamics [15,32–35] and MHD system [34–38], but the excessive dissipation of WENO scheme will cause underprediction of energy spectrum at small scales for smooth regions [39]. The hybrid approach can resolve the small scale motions and capture the strong continuities simultaneously, and has performed well in many benchmark tests [40–46] and shock-turbulence interaction problems [41,46–48] of hydrodynamic flows.

In this study, the hybrid compact-WENO scheme is developed for the compressible MHD system following the similar guidelines of Wang et al. [46]. The compact scheme and the WENO scheme are transited according to shock sensors proposed by Balsara and Spicer [49,50], which are the combination of criteria for large pressure jumps and strongly compressive regions. Based on these sensors, Balsara [51] also proposed a scheme, which preserved the positivity of density and pressure on very stringent problems fairly well. Indeed, they are helpful with intense turbulent simulations. In addition, we use a newly derived eighth-order pentadiagonal filter to eliminate spurious oscillations at high wavenumbers, following the approach proposed by Kim [28]. Spectral analysis shows that our new filter has a better performance and is more controllable than the filters proposed by Lele [16] and Gaitonde and Visbal [23]. To preserve the divergence-free condition of the magnetic field in numerical simulations, many strategies have been proposed, such as the projection method [52], the eight-wave formulation [6], the generalized Lagrange multiplier method [53], and the constrained transport (CT) method [49,54]. The staggered CT method requires the introduction of a staggered magnetic field variable, the interpolation of the face centered magnetic fields to the zone centers [34,35,38,55,56] and the construction of the electric field at the edges of the mesh [49,57,58]. It's proved to be accurate and efficient [59,60]. We adopt the unstaggered central difference type CT method of Tóth [59] to maintain the constraint in this work, which doesn't require spatial interpolation. Meanwhile, the projection method [59,60] is also applied to the magnetic field to eliminate the additional numerical errors that might lead to the violation of the divergence-free condition, such as those introduced by the filtering or forcing processes in the two-dimensional (2D) and three-dimensional (3D) simulations.

This paper is organized as follows. Section 2 gives the governing equations for the ideal and viscous MHD systems as well as the detailed numerical methods, including the hybrid compact-WENO scheme, the approaches to maintain the divergence-free property of the magnetic field, and the derivation of an eighth-order pentadiagonal compact filter. Section 3 presents two grid convergence tests to measure the numerical accuracy of the hybrid scheme, and a series of one-dimensional and two-dimensional benchmark tests to verify the capability of the hybrid scheme. Simulations of three-dimensional compressible MHD turbulence are also carried out and some statistical results are illustrated. The summary and some concluding remarks are finally drawn in Section 4.

2. Numerical method

2.1. Governing equations

The ideal MHD equations in the conservative form can be expressed as

$$\frac{\partial \boldsymbol{U}}{\partial t}+\frac{\partial \boldsymbol{E}}{\partial x}+\frac{\partial \boldsymbol{F}}{\partial y}+\frac{\partial \boldsymbol{G}}{\partial z}=0, \tag{1}$$

where $\boldsymbol{U}=(\rho, \rho u, \rho v, \rho w, B_x, B_y, B_z, \mathcal{E})^T$ is the vector of conservative variables, ρ is the density, u, v, w are the velocity components, B_x, B_y, B_z are the magnetic field components, $\mathcal{E}=\rho\boldsymbol{u}\cdot\boldsymbol{u}/2+p/(\gamma-1)+B^2/2$ is the total energy, p is the pressure, and γ is the adiabatic index. The inviscid fluxes $\boldsymbol{E}$, $\boldsymbol{F}$, $\boldsymbol{G}$ can be written as

$$\boldsymbol{E}=\begin{bmatrix}\rho u\\ \rho u^2+p_t-B_x^2\\ \rho uv-B_xB_y\\ \rho uw-B_xB_z\\ 0\\ -\Omega_z\\ \Omega_y\\ (\mathcal{E}+p_t)u-B_x(\boldsymbol{u}\cdot\boldsymbol{B})\end{bmatrix},\quad \boldsymbol{F}=\begin{bmatrix}\rho v\\ \rho vu-B_yB_x\\ \rho v^2+p_t-B_y^2\\ \rho vw-B_yB_z\\ \Omega_z\\ 0\\ -\Omega_x\\ (\mathcal{E}+p_t)v-B_y(\boldsymbol{u}\cdot\boldsymbol{B})\end{bmatrix},\quad \boldsymbol{G}=\begin{bmatrix}\rho w\\ \rho wu-B_zB_x\\ \rho wv-B_zB_y\\ \rho w^2+p_t-B_z^2\\ -\Omega_y\\ \Omega_x\\ 0\\ (\mathcal{E}+p_t)w-B_z(\boldsymbol{u}\cdot\boldsymbol{B})\end{bmatrix},$$

where $p_t=p+B^2/2$. The electric field vector is defined as $\boldsymbol{\Omega}=-\boldsymbol{u}\times\boldsymbol{B}$, and it is connected to the flux components by the following symmetric relations:

$$\Omega_x=-F_7=G_6,\quad \Omega_y=E_8=-G_5,\quad \Omega_z=-E_6=F_5. \tag{2}$$