Journal of Computational Physics 305 (2016) 185–207

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

Dynamic earthquake rupture simulations on nonplanar faults embedded in 3D geometrically complex, heterogeneous elastic solids

Kenneth Duru [a,*], Eric M. Dunham [a,b]

[a] Department of Geophysics, Stanford University, Stanford, CA, United States
[b] Institute for Computational and Mathematical Engineering, Stanford University, Stanford, CA, United States

A R T I C L E   I N F O

Article history:
Received 15 December 2014
Received in revised form 12 October 2015
Accepted 13 October 2015
Available online 19 October 2015

Keywords:
Earthquake rupture dynamics
Elastic waves
Nonlinear friction laws
Interface conditions
Interface waves
Curvilinear grids
Coordinate transformation
High order accuracy
Time-stability
Summation-by-parts finite difference method
Penalty method

A B S T R A C T

Dynamic propagation of shear ruptures on a frictional interface in an elastic solid is a useful idealization of natural earthquakes. The conditions relating discontinuities in particle velocities across fault zones and tractions acting on the fault are often expressed as nonlinear friction laws. The corresponding initial boundary value problems are both numerically and computationally challenging. In addition, seismic waves generated by earthquake ruptures must be propagated for many wavelengths away from the fault. Therefore, reliable and efficient numerical simulations require both provably stable and high order accurate numerical methods.

We present a high order accurate finite difference method for: a) enforcing nonlinear friction laws, in a consistent and provably stable manner, suitable for efficient explicit time integration; b) dynamic propagation of earthquake ruptures along nonplanar faults; and c) accurate propagation of seismic waves in heterogeneous media with free surface topography.

We solve the first order form of the 3D elastic wave equation on a boundary-conforming curvilinear mesh, in terms of particle velocities and stresses that are collocated in space and time, using summation-by-parts (SBP) finite difference operators in space. Boundary and interface conditions are imposed weakly using penalties. By deriving semi-discrete energy estimates analogous to the continuous energy estimates we prove numerical stability. The finite difference stencils used in this paper are sixth order accurate in the interior and third order accurate close to the boundaries. However, the method is applicable to any spatial operator with a diagonal norm satisfying the SBP property. Time stepping is performed with a 4th order accurate explicit low storage Runge–Kutta scheme, thus yielding a globally fourth order accurate method in both space and time. We show numerical simulations on band limited self-similar fractal faults revealing the complexity of rupture dynamics on rough faults.

* Corresponding author.
E-mail address: kduru@stanford.edu (K. Duru).

http://dx.doi.org/10.1016/j.jcp.2015.10.021

1. Introduction

Spontaneously propagating shear ruptures along frictional interfaces embedded in elastic solids occur in many solid mechanics and geophysical problems [51,21,49,31,44,2,41,46,48,45,47,55,4,3]. For example during an earthquake, two sides of the fault, initially held in contact by a high level of frictional resistance, slip suddenly when that resistance catastrophically decreases, generating strong ground shaking which is carried by seismic (elastic) waves to remote areas, far away from fault zones. Thus efficient and reliable simulations of earthquake ruptures and the resulting strong ground motion form a crucial component of earthquake-hazard analysis. The discontinuity in the tangential displacements across a fault zone is referred to as slip. Seismic waves generated by slip on one part of the fault transmit stresses to adjacent parts of the fault, possibly causing that part of the fault to slip and leading to the progressive propagation of ruptures. A typical dynamic earthquake rupture model couples the equations of linear elastodynamics to friction laws at the fault. The friction laws are generally expressed as nonlinear relations between tractions acting on the fault and the slip rate. The catastrophic initiation of slip and the sliding motion of the fault across the interface are captured via the interaction of friction and elastic waves. In dynamic earthquake rupture models, the earthquake source process is not known a priori, but is determined as part of the solution. The corresponding initial boundary value problem (IBVP) can be ill-conditioned, and poses several numerical and computational challenges. In addition, seismic waves generated by earthquake ruptures must be propagated many wavelengths far away from the fault. Therefore, reliable and efficient numerical simulations require both provably stable and high order accurate numerical methods.

A number of numerical methods have been developed for computational modeling of earthquake rupture dynamics. Classical finite difference methods (on planar faults) have been widely used [12,5,35,38]. Some of these methods have been extended to nonplanar fault geometries using curvilinear grids and coordinate transformation techniques [32], or by aligning the fault surface through cell diagonals [10,11]. However, the presence of nonlinear boundary and interface conditions, and discontinuities in media parameters make the design of stable and accurate finite difference methods challenging. The method developed in [32] for two space dimensional (2D) models can be proven stable; others produce solutions that appear accurate before eventually diverging, or require artificial dissipation of some form to stabilize the method. Another historical and popular class of methods for earthquake rupture dynamics is boundary element methods [46,22,48,2]. Thus far, these methods have been limited to uniform elastic materials. They can also handle nonplanar faults, with the exception of the spectral boundary integral equation method [22]. It is particularly noteworthy that spectral boundary integral equation methods are effective for planar fault problems, and have been extensively used to study fault weakening processes.

Unstructured discretizations such as continuous Galerkin finite element methods [25,34,1,14,18] and discontinuous Galerkin (dG) finite element/volume methods [13,42,56] have also been developed for earthquake rupture dynamics. The power of unstructured methods lies in their ability to resolve complex fault geometries. However, generation of a high quality unstructured mesh can be a non-trivial problem in itself. Unstructured discretizations also require extra bookkeeping and additional memory to keep track of the connectivity of the grid. Nonetheless, continuous Galerkin finite element methods are quite popular for rupture dynamics. In practice these methods have been limited to second order accuracy due to lumping of the mass matrix, which is necessary for computational efficiency. Since second order accuracy is not sufficient to propagate waves to remote areas, high order finite element methods have also been considered. Spectral element and dG finite element/volume methods [27,13,42,56] have been developed, yielding diagonal or locally block diagonal mass matrices. There are also potential drawbacks for these methods, due to doubling of the number of degrees of freedom along the element edges, and restrictive time-steps particularly for high order methods.

In this study, we develop a provably stable and high order accurate finite difference method for earthquake rupture dynamics on nonplanar faults embedded in three space dimensional (3D) geometrically complex, heterogeneous Earth models. In particular, we solve the first order form of the 3D elastic wave equation on a boundary-conforming curvilinear mesh, subject to traction free boundary conditions at the Earth's surface and nonlinear friction laws at the fault. The unknown field variables are particle velocities and stress fields that are collocated in space and time. Prior to numerical approximations, we transform the equations and boundary/interface conditions to computational cubes where numerical approximations of spatial derivatives can be performed efficiently. However, the coordinate transformation must be carefully introduced such that the transformed system of partial differential equations (PDEs) is numerically tractable. For linear hyperbolic problems in multiple dimensions, the SBP property is not sufficient for numerical stability [30,32]. This is because the way the metric terms enter the transformed system of PDEs and product rule does not explicitly hold for finite difference operators. This is truly a fundamental problem for numerical approximation of spatial derivatives on curvilinear meshes/elements [40,29,28,30,57,39,54,26,32]. Previous work has suggested to split the transformed equations into conservative and non-conservative parts such that the transformed equation is skew-symmetric [29,30,39,32]. This is critical for numerical stability. Unfortunately, splitting the equations increases the expected number of floating point operations. This is undesirable, particularly because of the sheer size of the problem in 3D. In this paper, we propose a new way to transform the equations of linear elastodynamics, which reduces the expected number of floating points operations at least by half while ensuring numerical stability. We use two different representations of the spatial derivatives: nonconservative form for the velocity fields and conservative form for the stress fields. This enables the construction of a continuous energy estimate, for the transformed equations, which can be mimicked to develop stable numerical approximations. After we have performed the coordinate transformation to the computational domain, the transformed equations are then discretized in space using high order accurate summation-by-parts (SBP) finite difference operators [53,36,23].