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Controlling bulk Reynolds number and bulk temperature in channel flow simulations

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ABSTRACT

Bulk Reynolds number and bulk temperature are key quantities when reporting results in channel flow simulations. There are situations when one wishes to accurately control these parameters while changing some numerical or physical conditions. A method to control the bulk Reynolds number and the bulk temperature in channel flow simulations is detailed. An ordinary differential equation is prescribed for the additional source term in the momentum balance equation so that the transient regime of the simulation is thoroughly tuned in order to efficiently and accurately reach the target Reynolds number value. A similar treatment is applied for the additional volume heat source term in the energy balance equation. The proposed method is specifically interesting when studying complex multi-physics in channel flow configurations when non-dimensionalization of the equations is no longer practical.

1. Introduction

Channel flow is a simple configuration that has been widely studied numerically in order to analyze turbulent boundary layers and validate models [1,2]. For fully developed channel flows, periodic conditions are considered in the infinite spanwise direction, and also along the streamwise direction. A homogeneous source term S_i is then added to the momentum equations to compensate for viscous forces and drive the flow at a given bulk velocity. In direct numerical simulations, the momentum equation in the streamwise direction ($i = 1$) reads

$$\frac{\partial}{\partial t}(\rho u_1) + \frac{\partial}{\partial x_j}(\rho u_j u_1) = -\frac{\partial p}{\partial x_1} + \frac{\partial \tau_{1j}}{\partial x_j} + S_1 \tag{1}$$

where S_1 is the homogeneous source term in the streamwise direction while $S_2 = S_3 = 0$. The source term S_1 enforces the flow mass flow rate and then determines the obtained Reynolds number. In standard flows with constant flow properties and without multiple physical phenomena (chemistry, radiation, ...), non-dimensional equations can be written and this source term is directly related to the intended bulk or friction Reynolds number. Alternatively, the source term can be simply updated step by step to compensate for the change of the intended mass flux [3]. However, in more complex flows featuring variable properties due to an explicit dependency on temperature for example [4], or involving multiphysics such as chemistry [5] or radiation [6,7], equations are kept in their dimensional form and S_1 must be determined differently to reach a target bulk Reynolds number Re_b^t.

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A first method to determine S_1 consists in taking a fixed constant in time S^{ref} [5,6] that is either chosen arbitrarily or more carefully evaluated from friction coefficient formulae that depend on the Reynolds number. Usually, the considered formulae are accurate for simple flows (constant properties, no multiphysics) but can become considerably erroneous as the studied flow is more and more complex. In such a case, the final Reynolds number that is obtained is different from the intended one.

A second kind of method consists in dynamically adapting the source term value after each iteration so that the Reynolds number is brought towards its target value [8,4,5,9]. However, none of these methods have been carefully characterized with a dedicated study. The procedures reported in [4,9] are similar to

$$S_1^{n+1} = S^{\text{ref}} + \frac{\rho_b^{\text{t}} u_b^{\text{t}} - \frac{\int_V \rho^n u^n \, dV'}{V}}{\tau^{\text{ref}}}, \tag{2}$$

where S_1^{n+1} is the updated value of the uniform momentum source term at iteration $n+1$, ρ_b^{t} is the target value of bulk density and u_b^{t} is the target value of bulk velocity. Equation (2) describes the adaptation of S_1^{n+1} because of the difference between the bulk mass flux at iteration n and its target value $\rho_b^{\text{t}} u_b^{\text{t}}$. This difference is related to the difference between the simulation bulk Reynolds number and the target value Re_b^t. The relaxation time τ^{ref} used in Eq. (2) is typically expressed in terms of the channel time scale δ/u_τ. When using Eq. (2), the permanent regime is reached after a transient stage and S_1^{n+1} tends towards a constant value denoted by $\overline{S_1}$, and

$$\rho_b^{\text{t}} u_b^{\text{t}} - \frac{\int_V \rho^n u^n \, dV'}{V} = \left(\overline{S_1} - S^{\text{ref}}\right) \tau^{\text{ref}}. \tag{3}$$

Since the obtained stationary value $\overline{S_1}$ is different from the empirically determined S^{ref} in complex flows, a finite bias between Re_b and Re_b^t unfortunately remains in such flows. This bias is not present in the formulation from Lenormand et al. [8] which can be reformulated as a PI controller with constant coefficients and a time response close to the computation time step. The efficiency of the approach was established *a posteriori*.

Equation (2) was thus a first attempt to control the bulk Reynolds number. This objective is here pursued by proposing an approach which ensures that the channel flow converges exactly and efficiently to the target bulk Reynolds number *a priori* and *a posteriori*. The approach appears as a modified PI controller with time varying coefficients. Carefully tuning the time varying coefficients allows to derive a second order ordinary differential equation with constant coefficients for Re_b whose time response can then be exactly controlled. The method is described in the following section. Its efficiency is then demonstrated in several channel flow configurations. Similarly, the method is derived and applied for the equivalent control of bulk temperature when studying turbulent heat transfer. Finally, a strategy for the control of both bulk Reynolds number and bulk temperature with variable thermo-physical properties accounted for is derived and validated in the section 4.

2. Control of the bulk Reynolds number

2.1. Formulation

Integration of Eq. (1) over the whole computational domain V gives

$$\frac{\text{d}}{\text{d}t}\left(\int_V \rho u_1 \, \text{d}V'\right) = \int \tau_{1j} n_j \, \text{d}S + S_1 V \tag{4}$$

where n_j is the outward surface normal vector. The integration of the pressure gradient and the convective terms is null because of the applied periodic boundary conditions in the X and Z directions.

The integrated term on the left side of Eq. (4) is related to the bulk Reynolds number $\text{Re}_b = \frac{\rho_b u_b \delta}{\mu_b}$,

$$\int_V \rho u_1 \, dV' = \rho_b u_b V = \frac{\mu_b \text{Re}_b V}{\delta} \tag{5}$$

where δ is the half-width of the channel and the subscript b is related to the aforementioned bulk quantities.

The integrated wall shear stress is split into two contributions,

$$\int \tau_{1j} n_j \, \text{d}S = -(\tau_{w,1} + \tau_{w,2}) S_w \tag{6}$$

as $\tau_{w,1}$ and $\tau_{w,2}$, the average wall shear stresses on the lower and upper wall respectively, can be different in the general case. S_w denotes the surface area of each wall.

Neglecting variations of the bulk dynamic viscosity and noticing that $V = 2\delta S_w$, Eq. (4) becomes

$$\frac{\text{dRe}_b}{\text{d}t} = -\frac{\tau_{w,1} + \tau_{w,2}}{2\mu_b} + \frac{\delta}{\mu_b} S_1. \tag{7}$$