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A stable fluid–structure-interaction solver for low-density rigid bodies using the immersed boundary projection method

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ABSTRACT

Dispersion of low-density rigid particles with complex geometries is ubiquitous in both natural and industrial environments. We show that while explicit methods for coupling the incompressible Navier–Stokes equations and Newton's equations of motion are often sufficient to solve for the motion of cylindrical particles with low density ratios, for more complex particles – such as a body with a protrusion – they become unstable. We present an implicit formulation of the coupling between rigid body dynamics and fluid dynamics within the framework of the immersed boundary projection method. Similarly to previous work on this method, the resulting matrix equation in the present approach is solved using a block-LU decomposition. Each step of the block-LU decomposition is modified to incorporate the rigid body dynamics. We show that our method achieves second-order accuracy in space and first-order in time (third-order for practical settings), only with a small additional computational cost to the original method. Our implicit coupling yields stable solution for density ratios as low as 10^{-4}. We also consider the influence of fictitious fluid located inside the rigid bodies on the accuracy and stability of our method.

1. Introduction

During recent years, the original immersed boundary (IB) method [1] has been extensively developed and gained popularity due to the ability to handle the interaction of objects of complex geometries with fluids. The key feature of the IB method is that the underlying Eulerian grid does not need to be body conforming. The application of the method ranges from fundamental problems of solid particle suspensions [2–4], to natural and industrial problems of complex and elastic geometries [5–10].

A number of studies [11–13] have reported difficulties with numerical convergence in fluid–structure interaction (FSI) problems, when the fluid force acting on the solid dominates over the solid inertial force, for example, in blood flow through flexible arteries, as discussed by Baek and Karniadakis [14]. Often the numerical instabilities are attributed to the added mass component. In the IB framework convergence problems have been reported by Borazjani et al. [13].

Numerical instabilities are also present in particulate flows. Uhlman [2] developed an efficient direct forcing IB method that describes the coupling between the Newton's equations of motion and the incompressible Navier–Stokes equations for many particles. However, due to the assumption that the fluid inside the particle moves as a solid body, the method suffers from stability issues for density ratio between solid and fluid below $\rho = \rho_s/\rho_f < 1.2$ (for spheres), where ρ_s is

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the density of the solid and ρ_f is the density of the fluid. The method was later improved by Kempe and Fröhlich [3] by taking the motion of the fluid inside the particle explicitly into account; they were able to reduce the critical density ratio to $\rho \approx 0.3$. Same approach was also used by Breugem [4]. All of these approaches use explicit coupling (so called weak coupling) between the rigid body and the fluid. The works by Uhlman [2], Kempe and Fröhlich [3] and Breugem [4] do not consider non-spherical bodies for which the fluid forces acting on the solid can be significantly larger. For such bodies, their algorithms are likely to be stable only for heavier particles with higher ρ.

Some studies have investigated the dynamics of more complex (non-spherical) particle geometries [13,15–17]. Zheng et al. [15] investigated human phonation numerically. To solve the motion of human vocal cords and interaction with surrounding air, they have developed a numerical method, in which a sharp-interface IB method is explicitly coupled with-finite difference Navier–Stokes solver. They show that the limiting density ratio, for which the method becomes unstable, is $\rho > 0.25$, which is similar as in the work by Kempe and Fröhlich [3]. They also have derived the necessary time step in order to resolve the motion of vocal cords and preserve stability. Borazjani et al. [13] compared weak coupling (WC) and strong coupling (SC) algorithms within the IB framework for different problems. The SC in their method is ensured using Gauss–Seidel-like iterations within each time step. They noticed that the WC algorithm becomes unstable, when the mass of the solid structure is reduced below some critical value. For certain problems they noticed that the SC algorithm also suffers from a similar drawback. A relaxation scheme for the inner iterations was implemented to overcome the problem, but with increased number of iterations and added computational cost.

Yang and Stern [16] have very recently presented a strongly-coupled non-iterative method using fractional step approach to solve Navier–Stokes equations coupled with sharp-interface direct-forcing IB method. They implemented a SC algorithm by introducing an intermediate step in a non-inertial reference frame, following the motion of solid body. Yang and Stern demonstrate improved stability properties with stable simulations down to density ratio $\rho \approx 0.1$, which is a significant improvement over work by Kempe and Fröhlich [3]. The derived formulation does not require any iterations within each time step and thus reduces the computational cost. Another non-iterative method is proposed by Gibou and Min [17], which is similar to algorithm described in the current paper. They advance both fluid and solid through intermediate states, and impose the interaction between fluid and solid during the projection step. In their work, the solution steps of the projection method for rigid body are deduced empirically by mimicking the fluid solution steps, while we use a more rigorous approach to derive solution steps for rigid body dynamics by a block-LU decomposition.

An alternative way to achieve better stability properties in simulations at low particle densities is including some information of added mass in the computational method. For example, Eldredge [18] has shown that the FSI computation can be stabilized if the added mass matrix is computed explicitly and added to the body inertia. Furthermore, Wang and Eldredge [19] have used some information about the added mass to arrive with relaxation factor, which leads to stable simulations.

In the present method, we discretize the system of equations following the approach by Taira and Colonius [20] and form a discrete linear system of equations. We then decompose the system using a block-LU decomposition, which gives us the prediction step for both fluid and solid body motion, the modified Poisson equation for the dynamic interaction force between fluid and solid, and the projection step for enforcing the interaction of the solid and fluid.

Wang and Eldredge [19] have developed a numerical method in which the null-space fluid solver of Colonius and Taira [21] is iteratively coupled with general equations for rigid body dynamics. The null-space based IB method [21] is an extension to the original IB projection method [20]. Our present FSI solver employs a direct solver for a positive-definite algebraic system, based on the block-LU decomposition in line with the original fractional step method [22], thus eliminating the need for any iterations within a single time step. This approach is illustrated using a special case of rigid body dynamics (non-deformable objects), while allowing extension to deformable, infinitely thin, open filaments and sheets. In addition, the current method gives direct access to the pressure field, which is useful in many applications.

We characterize the stability properties of our method on a vortex-induced-vibration (VIV) problem for two particles – a circular cylinder with and without a splitter plate clamped to the rear end. The current method is shown to be stable for solving the flow and body dynamics for both bodies for density ratios as low as 10^{-4}.

In section 2, we discuss the general governing equations for the physical problem of interest. In section 3, we describe the basic elements of the IB projection method by Taira and Colonius [20]. In section 4, we present our extension to the IB projection method for FSI problems with rigid bodies. We discuss Newton's equations of motion and couple them with the IB projection method using both explicit (WC) and implicit formulation (SC). We formulate the SC scheme in matrix form and decompose it using a block-LU decomposition. In section 5, we show the convergence properties of the current method. We also present results of a freely falling and rising circular cylinder and a neutrally buoyant circular cylinder in shear flow and compare our findings with literature. In section 6, we demonstrate the stability properties of our method for the VIV problem mentioned previously. In section 7, we investigate the effect of fictitious fluid inside the particle on numerical stability. Finally, we draw conclusions in section 8. In Appendix A, we present a modification of the present method, which is stable for limiting case of massless particles. In Appendix B, we show a design of a parallel algorithm for solution of Poisson equation, which does not depend on domain decomposition.