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A general solution strategy of modified power method for higher mode solutions

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ABSTRACT

A general solution strategy of the modified power iteration method for calculating higher eigenmodes has been developed and applied in continuous energy Monte Carlo simulation. The new approach adopts four features: 1) the eigen decomposition of transfer matrix, 2) weight cancellation for higher modes, 3) population control with higher mode weights, and 4) stabilization technique of statistical fluctuations using multi-cycle accumulations. The numerical tests of neutron transport eigenvalue problems successfully demonstrate that the new strategy can significantly accelerate the fission source convergence with stable convergence behavior while obtaining multiple higher eigenmodes at the same time. The advantages of the new strategy can be summarized as 1) the replacement of the cumbersome solution step of high order polynomial equations required by Booth's original method with the simple matrix eigen decomposition, 2) faster fission source convergence in inactive cycles, 3) more stable behaviors in both inactive and active cycles, and 4) smaller variances in active cycles. Advantages 3 and 4 can be attributed to the lower sensitivity of the new strategy to statistical fluctuations due to the multi-cycle accumulations. The application of the modified power method to continuous energy Monte Carlo simulation and the higher eigenmodes up to 4th order are reported for the first time in this paper.

1. Introduction

The power iteration method is widely used in nuclear criticality calculations, both with deterministic methods and Monte Carlo methods, to get the dominant eigenvalue and the corresponding eigenfunction. Interest in obtaining the higher mode eigenvalues and eigenfunctions is increasing, especially for reactor transient analysis, stability analysis, assessments of nuclear safety, as well as for the acceleration of source convergence. This can be done in various ways for deterministic calculations, and its efficiency can be very high. However, special attention needs to be paid to the Monte Carlo implementations.

The power iteration method is typically adopted in Monte Carlo criticality transport simulations. It is well known that after n iterations, the ratio of a higher mode to the fundamental mode will be $(|k_i|/k_0)^n$, where k_i is the ith mode eigenvalue, and the final results will be the fundamental mode after a sufficient number of iterations. That's the typical idea of criticality calculations with the Monte Carlo method.

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In the last few years, Booth proposed a modified power iteration method for simultaneously obtaining the first two eigenvalues and eigenfunctions, and it can accelerate the convergence of the fundamental mode (0th mode) [1–8]. This method directly subtracts out the 1st eigenmode and thus only powers out the second and higher eigenmodes. Booth and Gubernatis demonstrated the performance of the method using a one-dimensional one-group Monte Carlo neutron transport eigenvalue problem [6]. Shi and Petrovic implemented this method for one-dimensional two-group problems and proved its validity for these problems [9]. Yamamoto also studied the convergence of the 1st eigenmode in Monte Carlo power iteration using a one-dimensional two-group problem [10].

The modified power method can be generalized further to obtain the 2nd and even higher eigenmodes, but neither any practical solution nor any calculation application has been reported yet. Based on the newly formulated matrix form of the fission source transfer probabilities, a generalized solution strategy for simultaneously obtaining the first several higher eigenmodes is proposed in this work. It is demonstrated by the finite difference method (FDM) first, and then it is successfully implemented in the continuous energy Monte Carlo code.

There are some difficulties with the Monte Carlo implementation of the modified power method. One difficulty is that both positive and negative weights should be maintained and some weight cancellation scheme should be applied to estimate higher eigenmodes. Another difficulty is that for some cases the Monte Carlo implementation of the modified power iteration method may collapse due to some instability problems. The instability problem was reported by Booth and Gubernatis [6] and also mentioned in Yamamoto's paper [10]. It is caused by the statistical uncertainties inherent in Monte Carlo simulation. The accumulated tally technique is proposed in this work to overcome this problem.

2. Methodology

2.1. Review of modified power method

The modified power iteration method starts with two arbitrary functions, ψ_0 and ψ_1, both of which can be expressed as linear combinations of eigenfunctions:

$$\psi_0(r) = \sum_i a_i \phi_i(r), \qquad \psi_1(r) = \sum_i b_i \phi_i(r), \tag{1}$$

where $\phi_i(r)$ is the ith mode eigenfunction.

Updated solutions are calculated as a linear combination of $\psi_0(r)$ and $\psi_1(r)$ with parameter x:

$$\psi(r) = \psi_0(r) + x\psi_1(r) = \sum_i a_i \phi_i(r) + x \sum_i b_i \phi_i(r) = \sum_i (a_i + x b_i)\phi_i(r). \tag{2}$$

Suppose that after n power iterations, the higher eigenmodes powered out, leaving only the first two eigenmodes:

$$\begin{aligned} A^n \psi(r) &= \sum_i (a_i + x b_i) A^n \phi_i(r) = \sum_i (a_i + x b_i) k_i^n \phi_i(r) \\ &\approx k_0^n (a_0 + x b_0)\phi_0(r) + k_1^n (a_1 + x b_1)\phi_1(r), \end{aligned} \tag{3}$$

where **A** is the power iteration operator.

If $x = -a_1/b_1$, $\psi(r)$ will converge to the 0th eigenmode; while if $x = -a_0/b_0$, $\psi(r)$ will converge to the 1st eigenmode. The corresponding eigenvalue will be:

$$\lambda = \frac{\int \psi^n dr}{\int \psi^{n-1} dr} = \frac{\int A\psi^{n-1} dr}{\int \psi^{n-1} dr}, \tag{4}$$

which will converge to either the 0th or 1st eigenvalues, k_0 or k_1, depending on x.

Due to the characteristics of eigensystems, the integration in Equation (4) in any sub-region of the system must produce the same eigenvalue. If one chooses two sub-regions, R1 and R2, then Equation (4) can be written as:

$$\lambda = k = \frac{\int_{R1} \psi^n dr}{\int_{R1} \psi^{n-1} dr} = \frac{\int_{R2} \psi^n dr}{\int_{R2} \psi^{n-1} dr}, \tag{5}$$

and if V_{ij} and W_{ij} are defined as follows:

$$\int_{Rj} \psi_i dr = V_{ij}, \qquad \int_{Rj} A\psi_i dr = W_{ij}, \tag{6}$$

then Equation (5) can be written as:

$$\lambda = \frac{W_{01} + x W_{11}}{V_{01} + x V_{11}} = \frac{W_{02} + x W_{12}}{V_{02} + x V_{12}}, \tag{7}$$