Journal of Computational Physics 305 (2016) 664–676

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

Suppressing the numerical Cherenkov radiation in the Yee numerical scheme

Rachel Nuter *, Vladimir Tikhonchuk

University of Bordeaux, CNRS, CEA, CELIA, UMR 5107, F-33405 Talence, France

ARTICLE INFO

Article history:
Received 7 July 2015
Received in revised form 29 October 2015
Accepted 31 October 2015
Available online 4 November 2015

Keywords:
PIC code
Kinetic models
Collisionless shock
Numerical Cherenkov instability

ABSTRACT

The next generation of laser facilities will routinely produce relativistic particle beams from the interaction of intense laser pulses with solids and/or gases. Their modeling with Particle-In-Cell (PIC) codes needs dispersion-free Maxwell solvers in order to properly describe the interaction of electromagnetic waves with relativistic particles. A particular attention is devoted to the suppression of the numerical Cherenkov instability, responsible for the noise generation. It occurs when the electromagnetic wave is artificially slowed down because of the finite mesh size, thus allowing for the high energy particles to propagate with super-luminous velocities. In the present paper, we show how a slight increase of the light velocity in the Maxwell's equations enables to suppress this instability while keeping a good overall precision of calculations.

1. Introduction

The Yee Finite Difference Time Domain (FDTD) algorithm [1] is commonly used in PIC simulations for solving the Maxwell's equations. This scheme is easy to implement and robust for electromagnetic simulations. However, it suffers from a numerical dispersion in all propagation directions of electromagnetic waves except along the diagonal axes if the time step matches the upper limit of the Courant–Friedrichs–Lewy condition. This dispersion causes the numerical wave propagation speed to be slower than the physical wave propagation velocity. A significant error in the numerical results may be produced as high-energy particle velocity exceeds the numerical speed of electromagnetic waves. Such particles emit new electromagnetic waves leading to the so-called numerical Cherenkov instability [2]. It seriously affects multidimensional PIC simulations involving relativistic particles, such as for example, the laser-wakefield acceleration [3], streaming beams [4] and the collisionless shocks [5].

The Cherenkov radiation is a well-known physical process. It originates from high-energy particles moving with a velocity higher than the phase velocity of the light in a medium. It is responsible, for example, for the blue glow of an underwater nuclear reactor and widely used for diagnostic purposes. Because the particle velocity cannot exceed the light speed in vacuum (c), this process only occurs in gaseous, liquid or solid media, where the electromagnetic waves may travel with a phase velocity lower than c. In contrast, in fully ionized plasmas, the physical phase velocity of electromagnetic waves is higher than c, so that the Cherenkov radiation should not be observed. However, the numerical dispersion of the Yee scheme makes this process possible in plasma PIC simulations involving relativistic particles. In this case, the Cherenkov radiation is only a numerical artefact making simulation results noisy and with a lower accuracy. It is of particular importance for

* Corresponding author.
E-mail address: nuter@celia.u-bordeaux1.fr (R. Nuter).

http://dx.doi.org/10.1016/j.jcp.2015.10.057
0021-9991/

describing the laser-particle acceleration in a low density plasma, where such a noise affects the maximum attained energy and angular divergence of the particles [3]. It also modifies the collisionless shock propagation resulting from the interaction of two counter-propagating relativistic plasmas [5].

With the advent of ultra-high intensity lasers, relativistic particle beams are commonly generated. The numerical simulation results are then strongly affected by the generation of the numerical electromagnetic noise that, in turn, modifies the particle dynamics [3]. To inhibit this noise in PIC simulations, several numerical schemes have been proposed. Greenwood et al. [6] and Godfrey and Vay [7] implemented the spectral filtering. Even if this method is able to suppress the spurious radiation, it is not specifically selective to the Cherenkov emission. These authors have highlighted that the filter has to be carefully chosen in order to not suppress the real electromagnetic waves and alter the physics of interest. Vay et al. [8], Cowan et al. [9], Pukhov [10] and Lehe et al. [3] have proposed extended computational stencils for the numerical solution of the Maxwell–Faraday equations on a discrete grid. These numerical solvers, summarized in [11], allow to increase the numerical phase velocity along the grid axis so that the Cherenkov radiation is reduced. However, these algorithms need some extra numerical development and are still dispersive off the main axes.

In this article, we propose a new method to control and suppress the spurious Cherenkov instability. We keep the Yee FDTD algorithm to solve the Maxwell's equations, and do not use any extended computational stencil. We suggest to slightly increase the light speed in the Maxwell's equations while keeping it unaltered in the electron dynamic equations. With such a numerical method, the Cherenkov instability can then be suppressed along all directions, while not perturbing the particle dynamics.

The paper is organized as follows. In section 2 we present a numerical scheme for solving the Maxwell's equations and discuss its properties. We display the numerical dispersion relation, and show how one can control it with a numerical parameter modifying the light speed in vacuum. Section 3 details the numerical diagnostics considered to quantify the Cherenkov instability. Section 4 presents numerical simulations of a relativistic particle beam moving along the main propagation axis and demonstrates how the light speed increase enables to suppress the Cherenkov instability. Section 5 presents numerical simulations of the formation and propagation of a collisionless shock. In particular, we show how the spurious noise is deleted by increasing the light velocity without modifying the physical processes. We, finally, conclude in Section 6.

2. Yee FDTD and the dispersion formula

2.1. Maxwell's equations

Written in the International System of units, the Maxwell's equations are:

$$\frac{\partial \vec{B}}{\partial t} = -\vec{\nabla} \times \vec{E} \tag{1}$$

$$\frac{\partial \vec{E}}{\partial t} = \frac{1}{\mu_0 \epsilon_0} \vec{\nabla} \times \vec{B} - \frac{1}{\epsilon_0} \vec{j} \tag{2}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \tag{3}$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0} \tag{4}$$

where $\vec{E}$ is the electric field, $\vec{B}$ is the magnetic field, $\vec{j}$ is the current density, μ_0 is the magnetic constant and ϵ_0 is the electric constant. A solution of Eqs. (1) and (2) in vacuum ($\vec{j} = 0$), written in a form of a plane wave, $E_y(t - x/c)$, gives the relation $\mu_0 \epsilon_0 c^2 = 1$, where c is also the maximum speed of particles. Our method consists in decoupling the phase velocity of the light in vacuum, c_L, and the maximum particle velocity, c. More precisely, we are setting $\mu_0 \epsilon_0 c_L^2 = 1$, where c_L is the light speed which can be larger than c. Then, the modified Maxwell's equations are written as:

$$\frac{\partial \vec{B}}{\partial t} = -\vec{\nabla} \times \vec{E} \tag{5}$$

$$\frac{\partial \vec{E}}{\partial t} = c_L^2 \, \vec{\nabla} \times \vec{B} - \frac{1}{\epsilon_0} \vec{j} \tag{6}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \tag{7}$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0} \tag{8}$$

Note that inserting c_L into the Ampere's equation [Eq. (6)] does not modify the continuity equation as it concerns only the divergence-free field components. Indeed, combining Eqs. (6) and (8), one obtains:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \vec{j} = 0, \tag{9}$$

which is the standard form of the charge conservation equation.