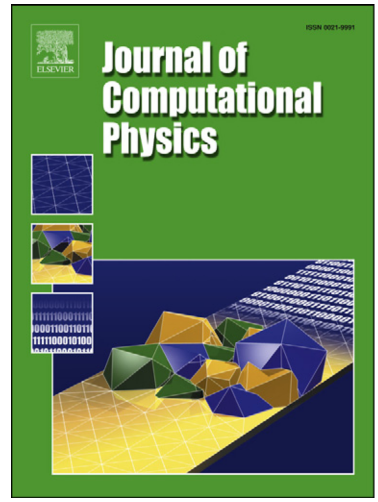

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A fourth-order accurate curvature computation in a level set framework for two-phase flows subjected to surface tension forces

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Abstract

We propose an accurate and robust fourth-order curvature extension algorithm in a level set framework for the transport of the interface. The method is based on the Continuum Surface Force approach, and is shown to efficiently calculate surface tension forces for two-phase flows. In this framework, the accuracy of the algorithms mostly relies on the precise computation of the surface curvature which we propose to accomplish using a two-step algorithm: first by computing a reliable fourth-order curvature estimation from the level set function, and second by extending this curvature rigorously in the vicinity of the surface, following the Closest Point principle. The algorithm is easy to implement and to integrate into existing solvers, and can easily be extended to 3D. We propose a detailed analysis of the geometrical and numerical criteria responsible for the appearance of spurious currents, a well known phenomenon observed in various numerical frameworks. We study the effectiveness of this novel numerical method on state-of-the-art test cases showing that the resulting curvature estimate significantly reduces parasitic currents. In addition, the proposed approach converges to fourth-order regarding spatial discretization, which is two orders of magnitude better than algorithms currently available. We also show the necessity for high-order transport methods for the surface by studying the case of the 2D advection of a column at equilibrium thereby proving the robustness of the proposed approach. The algorithm is further validated on more complex test cases such as a rising bubble.

Keywords: surface tension, curvature computation, level set method, spurious currents, two-phase flow, continuum surface force, balanced force algorithm, closest point method

1. Introduction

Surface tension is a key mechanical force in multi-phase flows where it plays a particularly important role in hydrodynamics at small scales. Rising bubbles and airborne water drops are common examples of such flows (see for example [1, 2, 3]). The surface tension force lies at the interface between the two phases and is thus singular at that location. This singularity makes it a complicated continuum mechanics problem when coupled with the Navier-Stokes equations.

From a numerical point of view, this singular force is challenging as it requires precise localization of the surface, its associated normal vector and curvature. A variety of methods have been introduced to localize the surface advected by the flow, the front-tracking method [4] in a Lagrangian framework, the Volume Of Fluid (VOF) method [5] and the Level Set (LS) method [6] in a Eulerian framework . These three widely used approaches offer advantages and drawbacks such that these methods are actively researched today. We base our original work on the level set framework which permits easy capture of topological changes and offers the advantage of simpler algorithms and a natural extension from 2D to 3D. However, in comparison to the Lagrangian representation, precise localization of the surface is lost and the level set method usually suffers from more volume loss than, for example, VOF methods. A wide range of techniques has been proposed to alleviate these problems such as the hybrid particle level set [7], the CLSVOF method [8] and high-order semi-Lagrangian particles method [9]. As we demonstrate in this article, accurate transport of the surface is required to attain high-order computation of the dependent curvature. In a Eulerian framework, the surface is immersed in the fluid through the use of a Dirac mass and a Heaviside function to characterize the phases.

The surface tension force is commonly introduced in the Navier-Stokes equations as an external force located at the interface. The Continuum Surface Force (CSF) model introduced in [10] gives an elegant solution to regularize the singular term. The method is based on spreading the force over a small neighborhood around the interface. In the last decade, research has been focused on the reduction of spurious currents, arising in the vicinity of the surface [11, 12, 13, 14, 15, 16]. These currents are numerical parasitic velocities introduced in the flow by errors in the estimation of the surface tension force. As demonstrated in [17], they are quadratically